

# Numerical Prediction of Humid Effect to Transonic Flows in Turbomachinery

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## ABSTRACT

Transonic condensate flows of moist air through cascade channel in turbomachinery are numerically predicted by using the high-resolution method developed by the authors. The governing equations consist of conservation laws of mixed gas, water vapor, water liquid, and the number density of water droplets, coupled with the momentum equations and the energy equation. The classical condensation theory is employed for modeling homogeneous nucleation and nonequilibrium condensation. These equations are solved by a high-order high-resolution finite-difference method based on the fourth-order compact MUSCL TVD scheme, Roe's approximate Riemann solver, and the maximum second-order LU-SGS scheme. Transonic condensate flows through a turbine and a compressor cascade channel are calculated with and without condensation. The calculated flow patterns and total pressure losses are compared with the experiments. These results indicate that the humid effect to turbomachinery cannot be neglected for evaluating the real performance in atmospheric humid conditions.

## INTRODUCTION

A number of numerical works for solving transonic flows in aircraft fan engines have been reported in last decades. However, all of them we know assume that gas in flows is dry air. Humid effects to turbomachinery have been neglected in them. However, the actual atmosphere of the earth includes a finite amount of water vapor. It plays an important role in weather conditions. Water vapor occasionally condenses around aircrafts which is cruising in a high humid condition.

Some research groups have calculated transonic wet-steam flows in steam turbine cascade channel considering condensation of water vapor[1][2]. Condensation observed in steam turbines is also of important phenomena in engineering. The phase change may be governed by homogeneous nucleation and the nonequilibrium process of condensation. The latent heat in water vapor is released to surrounding non-condensed gas when the phase change occurs, increasing temperature and pressure. This non-adiabatic effect induces a nonlinear phenomenon, the so-called "condensation shock". It is known that condensed water vapor

affects the performance of steam turbines. Transonic condensate flows of moist air around airfoil in atmospheric wind tunnel conditions have been also calculated by Schnerr and Dohrmann[3]. 3D flows around the ONERA M6 wing in the conditions have been solved by Yamamoto, Hagari and Murayama[4]. Condensate flows in atmospheric flight conditions over a 3D delta-wing have been calculated by Yamamoto[5], too. Those calculated results suggest that humid effects to the performance of wing cannot be neglected if we estimate the real performance in atmospheric humid conditions. Therefore, they suggest that transonic flows in turbomachinery may be also influenced by humid effects or condensation when turbomachine works in the humid conditions.

In this study, transonic condensate flows of moist air through a turbine and a compressor cascade channel are calculated by using the high-resolution method developed by our research group and calculated flow patterns and total pressure losses are compared with the experiments.

## Nomenclature

$C_p$	: Specific heat at constant pressure
$I$	: Nucleation rate
$T$	: Static temperature
$U_i$	: $\xi_i$ component of contravariant velocities
$R$	: Gas constant of two phase flow
$M$	: Molecular weight
$c$	: Speed of sound
$e$	: Total internal energy per unit volume
$n$	: Number density of water droplets
$p$	: Static pressure
$q_i$	: $\xi_i$ component of heat fluxes
$r$	: Averaged radius of droplet
$r_*$	: Critical radius of droplet
$t$	: Physical time
$u_i$	: $x_i$ component of physical velocities
$x_i$	: Component of Cartesian coordinates
$\Gamma$	: Mass generation rate of liquid phase

- $\beta$  : Condensate mass fraction  
 $\rho$  : Density  
 $\xi_i$  : Component of general curvilinear coordinates  
 $\tau_{ij}$  : component of viscous stress tensors

Subscripts

- $a$  : Dry air  
 $v$  : Water vapor  
 $l$  : Water liquid phase  
 $m$  : Two phase flow  
 $s$  : Saturation state

## FUNDAMENTAL EQUATIONS AND MODELS

The fundamental equations in this study consist of conservation laws of the total density, the momentums, the total energy, the density of water vapor, the density of water liquid, and the number density of water droplets in general curvilinear coordinates[4] as

$$\frac{\partial Q}{\partial t} + \frac{\partial F_i}{\partial \xi_i} + S + H = 0 \quad (1)$$

where,  $Q$ ,  $F$ ,  $S$ , and  $H$  are the vector of unknown variables, vector of flux, viscous terms, and source terms as follows.

$$Q = J \begin{bmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ e \\ \rho_v \\ \rho\beta \\ \rho n \end{bmatrix}, \quad F_i = J \begin{bmatrix} \rho U_i \\ \rho u_1 U_i + (\partial \xi_i / \partial x_1) p \\ \rho u_2 U_i + (\partial \xi_i / \partial x_2) p \\ (e + p) U_i \\ \rho_v U_i \\ \rho\beta U_i \\ \rho n U_i \end{bmatrix}$$

$$S = -J \frac{\partial \xi_j}{\partial x_i} \frac{\partial}{\partial \xi_j} \begin{bmatrix} 0 \\ \tau_{1i} \\ \tau_{2i} \\ \tau_{ik} u_k + q_i \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad H = -J \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -G \\ G \\ \rho I \end{bmatrix}$$

The equation of state, the speed of sound for moist air used in this study have been derived by Ishizaka, Ikohagi and Daiguji[6] assuming that the mass fraction of water liquid  $\beta$  is sufficiently small ( $\beta < 0.1$ ). These equations are given by

$$p = \rho RT(1 - \beta) \quad (2)$$

$$c^2 = \frac{C_{pm}}{C_{pm} - (1 - \beta)R} \frac{p}{\rho} \quad (3)$$

where,  $C_{pm}$  is defined by the linear combination of the specific heat at constant pressure between gas phase and liquid phase using the mass fraction  $\beta$ .

The mass generation rate  $\Gamma$  for water droplets is formed as a sum of the mass generation rate of critical-sized nucleus and the growth rate of a water droplet based on the classical condensation theory and it is further approximated by Ishizaka et al.[6] as,

$$\Gamma = \frac{4}{3} \pi \rho_l I r_*^3 + 4 \pi \rho_l n r^2 \frac{dr}{dt} \quad (4)$$

where, the homogeneous nucleation rate  $I$  has been defined by Frenkel[7]. The growth rate of a water droplet  $dr/dt$  is given by Hertz-Knudsen model. The approximate model proposed by Schnerr and Dohrmann[3] is used.

## NUMERICAL METHOD

The high-order high-resolution finite-difference method based on the fourth-order MUSCL TVD scheme[8] and the Roe's approximate Riemann solver[9] is used for space discretization of convection terms in the fundamental equations Eq.(1). The viscosity term is calculated by the second-order central-difference scheme. The maximum second-order LU-SGS method based on the Newton-iteration and the Crank-Nicolson method is used for the time integration[10]. The Baldwin-Lomax model[11] is used for evaluating eddy-viscosity.

## RESULTS

### Transonic condensate flow of moist air in Laval Nozzle

A transonic flow in two-dimensional Laval nozzle is first calculated considering homogeneous nucleation and nonequilibrium condensation. The calculated pressure distributions are compared with experiments[12]. A two-dimensional computational grid is generated. The grid points are 151x51. As flow conditions, the inlet stagnation pressure is  $1.0 \times 10^5$ [Pa], the inlet stagnation temperature is 296.6[K] and the stagnation relative humidity is 36.4[%].

Figure 1 shows the calculated pressure coefficient distributions along the symmetric line in the nozzle. The calculated results considering condensation are compared with the experiments and those without condensation. In this case, the condensation coefficient in the nucleation rate  $I$  is set at 0.3 to get the best fitted results with the experiments. Then, the present results are in good agreement with the experiments.

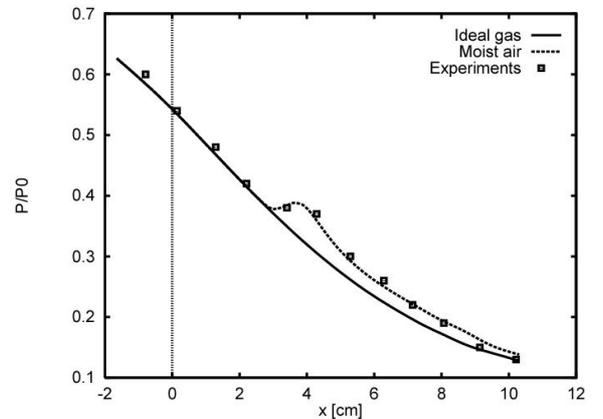


Fig. 1 Pressure distributions in Laval nozzle

Figure 2 shows the calculated contours of condensate mass fraction. Condensation occurs in the flow expansion after the nozzle throat. This condensation releases latent heat due to phase change from water vapor to water liquid. It results in pressure increase as shown in Fig.1. The condensation model used in this study is well tuned for homogeneous condensation by this calculation.

**Transonic and supersonic condensate flows of moist air in turbine cascade channel**

The present method is applied to transonic and supersonic flows through a two-dimensional turbine cascade channel in atmospheric wind-tunnel conditions with and without condensation, which experimental results have been reported by Nagayama et al. [13] to validate the present numerical method and the condensation model.

Figure 3 shows the modified H-type computational grid we used. The grid points are 197×64. The flow conditions are that the stagnation pressure is  $1.0 \times 10^5$ [Pa], the stagnation temperature is 298[K], and the stagnation relative humidity is 80%. The initial inlet Mach number set at 0.2. Three cases such as the isentropic exit Mach number at 0.8(CASE A), 1.1(CASE B), and 1.6(CASE C) reported in Ref.[13] are taken into account.

Figures 4(a), (b) and (c) show each comparison between the experimental photograph of interference and the corresponding calculated pattern. In Fig. 4(a) for CASE A, the calculated pattern is well compared with the experiment. No shocks are observed in both results. Condensation doesn't occur in this case. The calculated maximum Mach number in flow is 0.91, while it is 0.89 in the experiment. With respect to Fig. 4(b) and 4(c) for CASE B and C, Nagayama et al.[13] reported that shocks are observed and condensation occurs in those cases. Shocks are also observed in the

calculated results and flow patterns are in good agreement with the experiments.

Figures 5 and 6 show the calculated contours of nucleation rate and those of condensate mass fraction in CASE B. Nucleation starts rapidly in the cascade channel due to homogeneous nucleation after the excessive supercooling of water vapor. A huge number of nuclei are produced by the homogeneous nucleation and each nucleus grows through the cascade channel. Water droplets

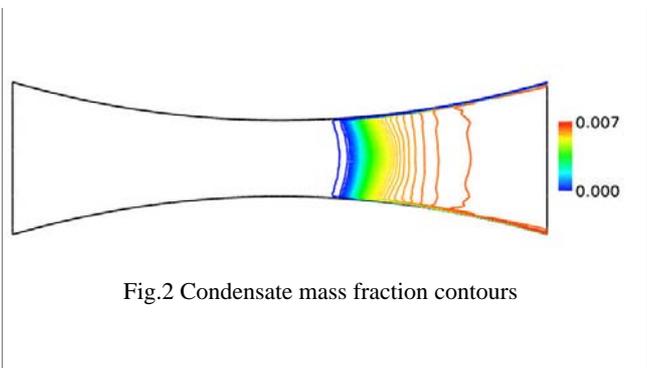


Fig.2 Condensate mass fraction contours

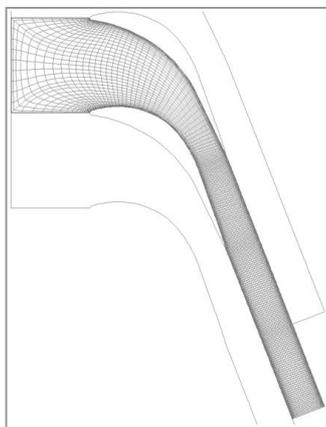


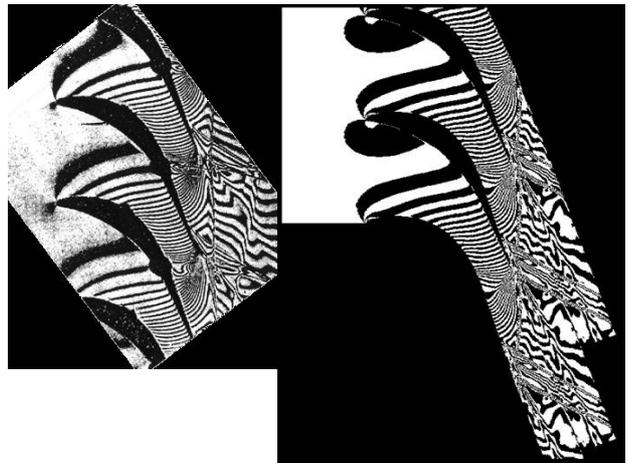
Fig. 3 Computational grid



(a) CASE A



(b) CASE B



(c) CASE C

Fig. 4 Density distributions (Left: experiment, Right: calculation)

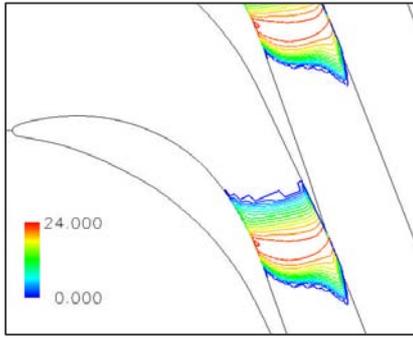


Fig.5 Nucleation rate contours(CASE B)

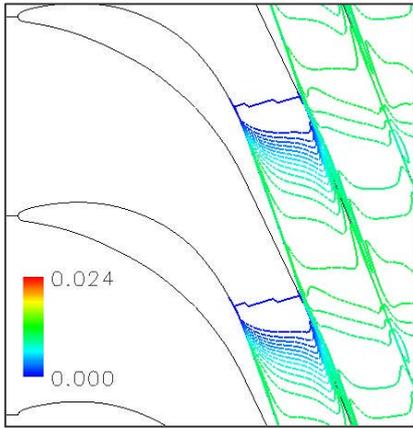


Fig.6 Condensate mass fraction contours (CASE B)

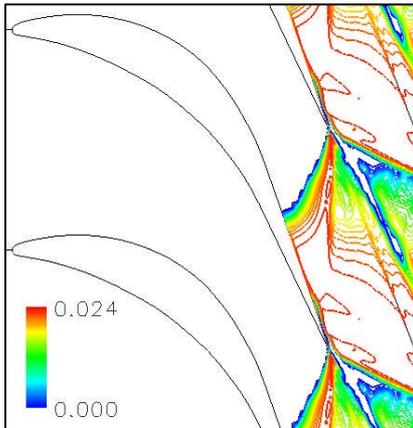


Fig. 7 Condensate mass fraction contours (CASE C)

are found after the throat of the cascade channel and they stream downward. The maximum rate of the mass fraction is sufficiently smaller than 0.1. The calculated contours of condensate mass fraction in CASE C is also plotted in Fig.7. In this case, oblique shocks are produced from the trailing edge of the turbine blade. The experimental photos in Ref.[13] indicate that the shock patterns are not so different from the case in dry air conditions even if condensation occurs in humid conditions. We can get almost a same flow pattern with and without condensation, too. However, a finite amount of condensation is captured by the calculation as shown in Fig.7.

Total and static pressure distributions at exit in dry and humid conditions reported in Ref.[13] are recreated by the present calculation. The flow conditions specified here is the same with the

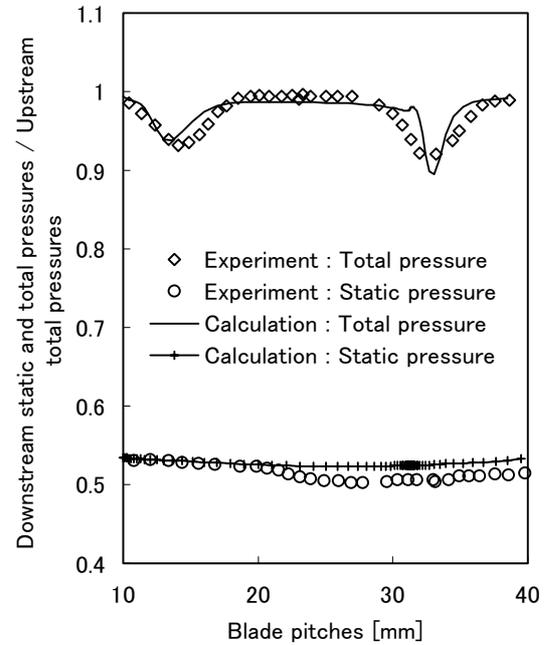


Fig.8 Total and static pressure distributions (dry air)

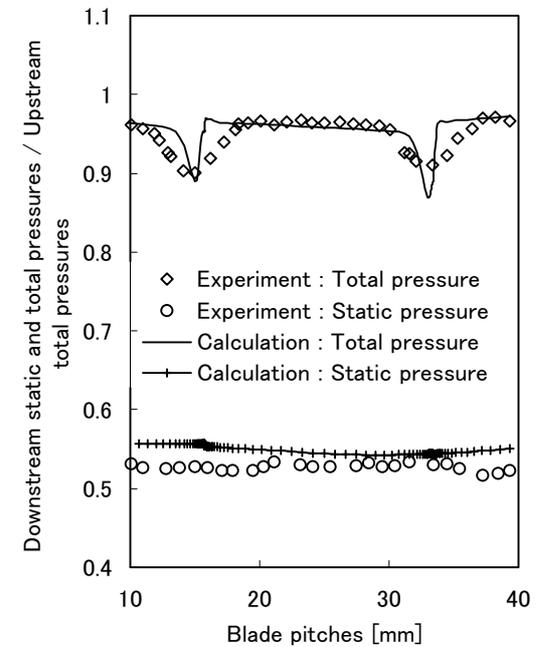


Fig. 9 Total and static pressure distributions (moist air)

previous condition except for the isentropic exit Mach number 1.2.

Figures 8 and 9 show the calculated total and static pressure distributions compared with the experiments in dry and humid conditions. The comparison of the experiments indicates that the total pressure is lower and the static pressure is slightly higher in humid conditions than those in dry air conditions. Same tendencies can be also demonstrated by the calculation, though some discrepancies between the calculations and the experiments are locally observed in Figs. 8 and 9. These results suggest that condensation in atmospheric humid conditions may unexpectedly affect the performance of turbo-machine. The averaged total pressure loss in the case of dry air is 0.981, while that of moist air is 0.959. The difference between them is believed to be mainly due to the effect of condensation.

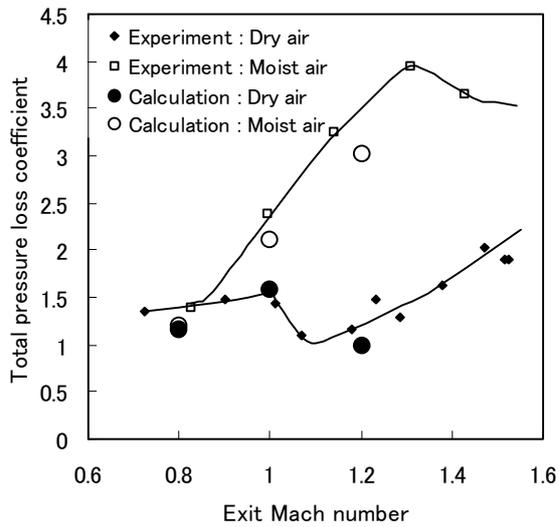


Fig. 10 Total pressure loss coefficients

The dependence of the isentropic exit Mach number to the total pressure loss is next examined. The averaged total pressure loss coefficients calculated using the conditions of the isentropic exit Mach number at 0.8, 1.0, and 1.2 with and without condensation are plotted in Fig. 10 and compared with experiments by Nagayama et al.[13]. These values are normalized by the smallest value obtained by the case at the exit Mach number 1.17 in experiments and that at 1.20 in calculations. In the experiments, the loss in the case with condensation is drastically increased as increasing in the exit Mach number compared with that in the case without condensation. The calculated values can demonstrate these tendencies.

### Transonic condensate flows in compressor cascade channel

Since inlet temperature may be sufficiently high in actual gas-turbine cascades because of combustion heating, condensation may be neglected. On the other hand, since atmospheric air passes through fan rotors or compressors in aircraft fan engine, condensation must be presented in these elements if it is in humid conditions. Unfortunately, we've had no experimental data for such condensation. Therefore, transonic flows in a compressor cascade channel are numerically and preliminary predicted assuming that the present numerical method can be applied also to the problems for transonic condensate flows in compressors.

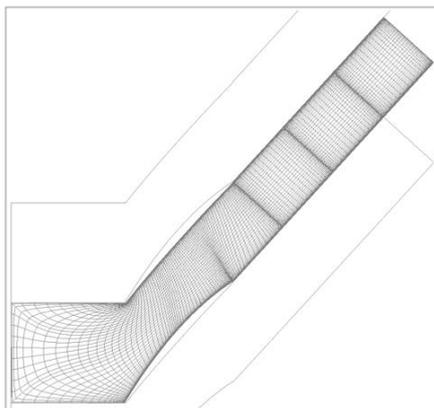
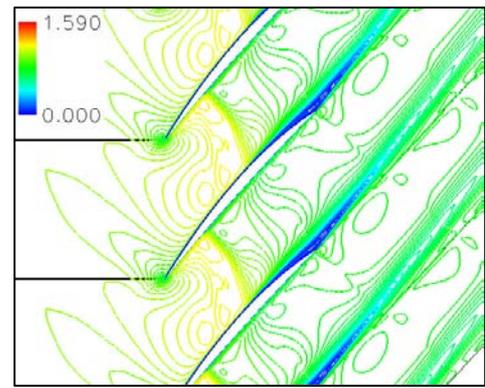
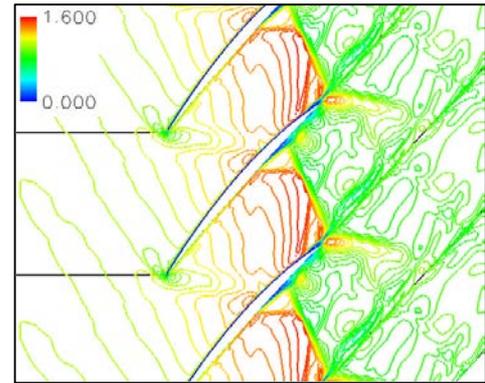


Fig. 11 Computational grid



(a) Dry air



(b) Moist air

Fig. 12 Instantaneous Mach number contours

Figure 11 shows the modified H-type computational grid with  $175 \times 64$  grid points and the blade shape[14]. The flow conditions are that the stagnation pressure is  $1.0 \times 10^5$  [Pa], the stagnation temperature is 298 [K], the Reynolds number is  $1.6 \times 10^6$ , and the stagnation relative humidity is 80%. A well-known case for this cascade such as the inlet flow angle at  $56.8^\circ$ , the inlet Mach number fixed at 1.02, and the static pressure ratio at 1.34 is taken into account.

Figures 12(a) and 12(b) show the calculated Mach number contours in dry air and humid conditions. A typical flow pattern can be found in Fig.12(a), in which a detached bow shock and a normal shock are observed. On the other hand, the normal shock in Fig.12(a) is shifted downward when it is in humid conditions as

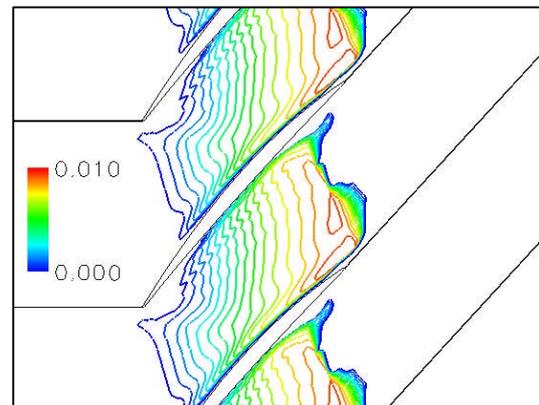


Fig. 13 Instantaneous contours of condensate mass fraction

shown in Fig.12(b). Since condensation releases latent heat due to phase change from water vapor to water liquid, temperature and pressure are increased. The pressure increase results in the shock movement downward. Similar characteristics have been already reported in the previous work[4]. An additional interesting point in this case for the compressor is that condensation strengthens the normal shock. Compressor rotors have strong adverse gradient in pressure. Pressure increase due to the release of latent heat may encourage the adverse pressure gradient. Fig.13 shows the calculated condensate mass fraction contours in the humid conditions. Onset of condensation is located at the inlet of the compressor cascade channel and condensed water liquid is soon evaporated after the normal shock because of rapid increase in pressure.

## CONCLUDING REMARKS

Transonic condensate flows through a turbine cascade channel were calculated considering atmospheric humid conditions and condensation. The calculated flow patterns and total pressure losses were well compared with the experiments. The computational code was also extended to a transonic compressor case. In this case, condensation strengthened the normal shock in the cascade channel because of the adverse gradient of pressure in compressor. These calculated results and the corresponding experiments suggest that the humid effect to the performance of turbo-machine cannot be neglected for evaluating the real performance in atmospheric humid conditions.

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