

Reduction of Vibratory Stress of Compressor Blade by Use of Asymmetric Vane Spacing

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ABSTRACT

It is well known that asymmetric vane spacing can result in decreased levels of the excitation at specific frequencies. In this paper, the resonant response reduction of compressor blades due to asymmetric vane spacing is studied both theoretically and experimentally for the most probable asymmetric vane in which the vane count of the upper and lower half is slightly different. First, a method for predicting the vibratory stress of the blade for the asymmetric spacing vane is proposed. Secondly, using a simple model of the asymmetric vane, a parametric study is carried out to clarify the influence of blade damping and vane count on the blade resonant response reduction. Finally, experiments are carried out to verify the validity of the proposed method and the effect of asymmetric vane spacing on resonant stress response using a 3 stage scaled-model compressor.

INTRODUCTION

In a multi-stage turbo-machinery, the interaction between the vane and the blade generates the excitation force on the blade, which comes from the wake of the upstream vane or the potential field of the upstream/downstream vane. The fundamental frequency of the excitation force due to the interaction between the vane and the blade is the rotor speed multiplied by the vane count, and if the natural frequency of the blade is coincident with the frequency of the excitation force, the resonant stress of the blade may become very large and may cause a blade failure due to HCF. Traditionally, the blade designer has adopted the following methods to reduce the probability of failure:

- (1) Avoid the resonance by detuning the blade frequency or altering the vane count (Srinivasan, 1997).
- (2) Reduce the excitation force by expanding the spacing between the vane and the blade (Gallus, 1982, and Hoyningen-Huene and Hermeler, 1999) or by use of clocking (Hsu and Wo, 1998 and Benini and Toffolo, 2002).
- (3) Increase the blade damping by adopting the friction damper, etc. (Griffin and Labelle, 1996, and Panning, 2002).

Besides these methods, it is well known that the asymmetric vane spacing can result in decreased levels of the excitation force at specific frequencies (Kemp, 1958, and Clark, 2002). It seems that asymmetric vane spacing is a very effective method for controlling the blade response for variable speed engines

where the axial rotor span cannot be lengthened.

When applying asymmetric vane spacing to actual engines, it is essential to be able to predict the response reduction expected relative to symmetric vane spacing. Blade designers have to judge whether HCF blade failures that occur with symmetric vane spacing can be prevented with asymmetric vane spacing. As far as the authors know, the primary analytical method for predicting the resonant response reduction is Fourier analysis. With this method, the circumferential distribution of the vane wakes is analyzed by Fourier analysis to evaluate the magnitude of the excitation force at specific frequencies. The deficiency of this method is that the effects of damping on the transient response are not included and the Fourier analysis overpredicts the response reduction observed.

This paper proposes a practical analysis method for predicting the response reduction due to asymmetric vane spacing using 3-D CFD and the modal analysis method based on 3-D FEM. With this method, a time-history wave of vibratory stress is predicted for both the symmetric and asymmetric vane spacings. Also included in this paper is a parametric study using a simple model of the asymmetric vane spacing to examine the effect of the vane count and the blade damping on the blade vibratory stress. Finally, a rig test was carried out where both symmetric and asymmetric vane spacings were tested. Measured blade vibratory stresses were compared to predicted values using the proposed analysis method. The validity of the proposed method is verified as well as the reduction effect on vibratory stress due to asymmetric vane spacing.

ANALYTICAL METHOD

As for the asymmetric vane spacing, usually whole vanes in the stage are equally divided into N segments, and the following techniques are used to make asymmetric vane spacing.

- (1) Vane count in N segments is slightly altered (Vane count method).
- (2) Vane count in N segments is equal but the vane pitch between segments is changed by shifting segments circumferentially with respect to each other (Tangential offset method).
- (3) All vane pitches are altered at random (Random pitch method).

Although these methods are well known, it seems that only the vane count method, where the whole vanes are divided into two segments (N = 2, upper half and lower half) and the vane count in the upper and lower half is slightly altered, is most

practical and has been applied to actual engines, taking the cost of manufacturing and maintenance, disadvantageous effect on the excitation force of the lower engine orders, and the stability of the flow such as surge into consideration. Therefore, in this paper, the analytical method for predicting the blade vibratory stress of the asymmetric vane spacing by use of vane count method ($N = 2$) is described, but the same procedure can be used for the offset method.

Figure 1 shows the procedure for calculating resonant stress of the blade caused by the excitation force due to the asymmetric vane spacing, where the vane counts of the upper and lower half are N_1 and N_2 , respectively. First, 3-D CFD analysis is carried out to obtain the time-resolved flow field for 2 kinds of symmetric vane spacings, where the total vane count of the stage are $2N_1$ and $2N_2$. The modal force and resonant stress for the symmetric vane spacing is calculated by using the results of 3-D CFD and 3-D FEM analysis. Secondly, the modal analysis technique is used to represent the vibration characteristics of each mode by the spring-mass system with 1 degree of freedom. The vibratory response of the blade for the asymmetric vane spacing is calculated by the transient time-history response analysis of the blade passing through the upper and lower half with the different vane count. The reduction effect of the vibratory stress is obtained by comparing the response for the asymmetric vane spacing with that for the symmetric vane spacing. This method of analysis seems to be somewhat less rigorous than a true full stage calculation because the excitation force is generated by combining two kinds of time-history waves of the pressure fluctuation, and therefore the discontinuity between the upper and lower half cannot be considered exactly. However, this method can be applied to predict the vibratory stress of the blade for the asymmetric vane spacing, for the input energy to the blade from around the boundary of the segments is much smaller than that from the whole stage. The detailed procedure for calculating the vibratory stress of the blade is described in the next section.

Calculation of resonant stress for symmetric vane spacing

Figure 2 shows the procedure for calculating the resonant stress of the blade for a symmetric vane spacing. This method is the almost same one as many published papers adopted

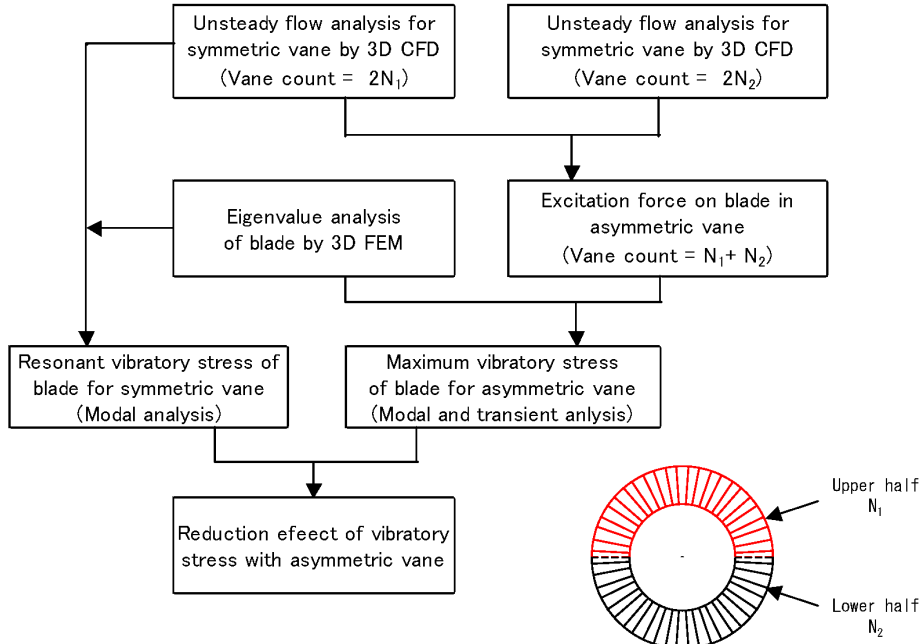


Fig. 1 Procedure for calculating vibratory stress for asymmetric vane spacing

(Chiang and Kielb, 1993, Hilbert, 1997, and Filsinger, 2001), and the resonant vibratory stress is calculated as follows.

The pressure fluctuation on the blade surface is first obtained in the form of time-history wave by the blade-vane interaction analysis with 3-D CFD. The magnitude and phase of the pressure fluctuation with the fundamental frequency (rotor speed multiplied by the vane count) is calculated by Fourier analysis. Next, the eigenvalue analysis of the blade is carried out by 3-D FEM, and the resonant stress is calculated by the modal analysis method. Excitation force on each grid point of FE model is obtained by the pressure fluctuation calculated by 3-D CFD. In brief, the resonant stress of the blade for the symmetric vane spacing can be calculated from Eq.(1) and Eq.(2).

$$\sigma_{s,n} = \frac{\pi}{\delta_n} \frac{F_n}{m_n \omega_n^2} \sigma_{rel,n} \quad (1)$$

$$F_n = \left\{ \left(\sum_k \Delta_k \cos \alpha_k \right)^2 + \left(\sum_k \Delta_k \sin \alpha_k \right)^2 \right\}^{1/2} \quad (2)$$

$$\Delta_k = C_k \left(A_k^x \phi_k^x + A_k^y \phi_k^y + A_k^z \phi_k^z \right)$$

where,

- $\sigma_{s,n}$: Resonant stress for symmetric vane spacing
- $\sigma_{rel,n}$: Relative stress calculated by FEM
- F_n : Modal force
- m_n : Modal mass
- δ_n : Logarithmic decrement
- ω_n : Natural frequency of blade
- C_k : Magnitude of pressure fluctuation with fundamental frequency
- α_k : Phase of pressure fluctuation with fundamental frequency
- $\phi_k^x, \phi_k^y, \phi_k^z$: Displacement in x, y, z direction at k-th node
- A_k^x, A_k^y, A_k^z : Effective area in x, y, z direction at k-th node
- suffix n : Index of n-th mode
- suffix k : Index of k-th node in FE model

Modal logarithmic decrement in Eq.(2) consists of material, structural and aerodynamic damping and usually it is difficult

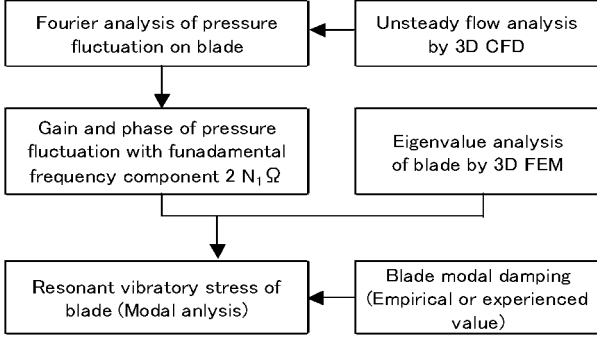


Fig. 2 Procedure for calculating vibratory stress for symmetric vane spacing

to estimate it by analytical method. Therefore empirical or experienced value is used for the modal logarithmic decrement.

Calculation of resonant stress for asymmetric vane spacing

Figure 3 illustrate the typical vibratory response of the blade passing through the asymmetric vane spacing when the blade resonate with rotor speed multiplied by twice the vane count of the upper half. Assuming that the rotational speed of the rotor, Ω is constant, the motion of the blade passing through the asymmetric vane spacing, where the vane counts of the upper and lower half are N_1 and N_2 , is governed by the following equation.

$$m_n \ddot{x}_n(t) + c_n \dot{x}_n(t) + k_n x_n(t) = \begin{cases} F_{n1} \sin \omega_1 t & \text{(in upper half)} \\ F_{n2} \sin \omega_2 t & \text{(in lower half)} \end{cases} \quad (3)$$

Where,

- m_n : Modal mass
- c_n : Modal damping coefficient
- k_n : Modal stiffness
- x_n : Generalized coordinate
- F_{n1} : Modal force of the blade passing through the upper half
- F_{n2} : Modal force of the blade passing through the lower half
- ω_1 : Excitation frequency in the upper half ($2N_1\Omega$)
- ω_2 : Excitation frequency in the lower half ($2N_2\Omega$)
- N_1, N_2 : Vane counts of the upper and lower half
- t : Time

Modal forces in the right hand of Eq.(3) can be obtained from 3-D CFD and 3-D FEM analysis for two kinds of the symmetric vane spacings, where the vane counts of the whole stage are $2N_1$ and $2N_2$, as explained in the previous section. Although the suffix n in Eq.(3) indicates the n -th vibration mode, the suffix n is dropped in the latter equation for

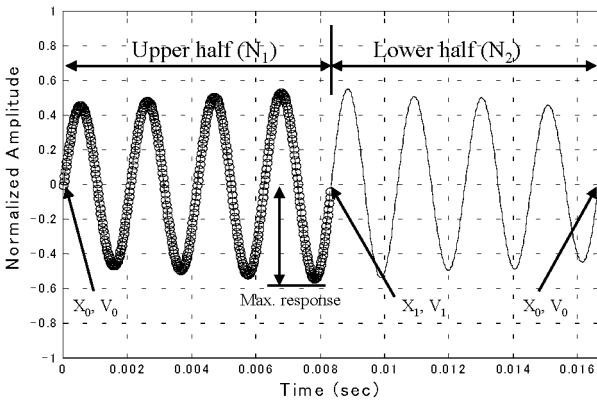


Fig. 3 Typical vibratory response of the blade passing through the asymmetric vane spacing

simplicity, unless confusion occurs.

When the blade is passing through the upper half, the solution of Eq.(3) can be expressed by Eq.(4).

$$x(t) = (x_0 - x_{s1}R_{dc1})u_1(t) + (v_0 - \omega_1 x_{s1}R_{ds1})u_2(t) + x_{s1}R_{dc1} \cos \omega_1 t + x_{s1}R_{ds1} \sin \omega_1 t \quad \text{(in upper half)} \quad (4)$$

Where,

$$x_0 = x(0), \quad v_0 = \dot{x}(0)$$

$$p_1 = \frac{\omega_1}{\omega_n}, \quad \omega_n = \sqrt{k/m}, \quad \zeta = \frac{c}{2\sqrt{mk}}, \quad x_{s1} = \frac{F_1}{k}$$

$$R_{dc1} = \frac{1-p_1^2}{(1-p_1^2)^2 + 4\zeta^2 p_1^2}, \quad R_{ds1} = \frac{2\zeta p_1}{(1-p_1^2)^2 + 4\zeta^2 p_1^2}$$

$$u_1(t) = e^{-\zeta\omega_n t} \left(\cos \omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin \omega_d t \right)$$

$$u_2(t) = \frac{1}{\omega_d} e^{-\zeta\omega_n t} \sin \omega_d t$$

$$\omega_d = \sqrt{1-\zeta^2} \omega_n$$

By substituting $t = T/2$ ($T = 2\pi / \Omega$: period of rotation) into Eq.(4) and another equation which can be obtained by differentiating Eq.(4) with respect to time t , displacement x_1 and velocity v_1 at the moment when the blade arrives the lower half are calculated in the form of Eq.(5).

$$\begin{cases} x_1 = a_1 x_0 + b_1 v_0 + c_1 \\ v_1 = a_2 x_0 + b_2 v_0 + c_2 \end{cases} \quad (5)$$

Where,

$$a_1 = e^{-y_1} \left(\cos z_1 + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin z_1 \right)$$

$$b_1 = \frac{1}{\omega_d} e^{-y_1} \sin z_1$$

$$c_1 = -x_{s1}R_{dc1}a_1 - \omega_1 x_{s1}R_{ds1}b_1 + x_{s1}R_{dc1}$$

$$a_2 = -\zeta\omega_n a_1 + \omega_d e^{-y_1} \left(\frac{\zeta}{\sqrt{1-\zeta^2}} \cos z_1 - \sin z_1 \right)$$

$$b_2 = e^{-y_1} \left(\cos z_1 - \frac{\zeta\omega_n}{\omega_d} \sin z_1 \right)$$

$$c_2 = -x_{s1}R_{dc1}a_2 + x_{s1}R_{ds1}\omega_1(1-b_2)$$

$$y_1 = \frac{2\zeta N_1 \omega_n \pi}{\omega_1}, \quad z_1 = \frac{2\omega_d N_1 \pi}{\omega_1}$$

By substituting x_1 , v_1 and $t-T/2$ for x_0 , v_0 , and t in Eq.(4), the response of the blade passing through the lower half can be expressed by Eq. (6).

$$x(t) = (x_1 - x_{s2}R_{dc2})u_1\left(t - \frac{T}{2}\right) + (v_1 - \omega_2 x_{s2}R_{ds2})u_2\left(t - \frac{T}{2}\right) + x_{s2}R_{dc2} \cos \omega_2 t + x_{s2}R_{ds2} \sin \omega_2 t \quad \text{(in lower half)} \quad (6)$$

Where,

$$p_2 = \frac{\omega_2}{\omega_n}, \quad x_{s2} = \frac{F_2}{k}$$

$$R_{dc2} = \frac{1-p_2^2}{(1-p_2^2)^2 + 4\zeta^2 p_2^2}, \quad R_{ds2} = \frac{2\zeta p_2}{(1-p_2^2)^2 + 4\zeta^2 p_2^2}$$

In the steady state, the displacement and velocity of the blade arriving at the upper half, which can be obtained by substituting $t = T$ into Eq.(6) and its differentiation, should be equal to x_0 and v_0 in Eq.(3). To meet this prerequisite condition, Eq.(7) should be satisfied.

$$\left. \begin{aligned} x_0 &= a_1 x_1 + b_1 v_1 + c_3 \\ v_0 &= a_2 x_1 + b_2 v_1 + c_4 \end{aligned} \right\} \quad (7)$$

Where,

$$\begin{aligned} c_3 &= -x_{s2} R_{dc2} a_1 - \omega_2 x_{s2} R_{ds2} b_1 + x_{s2} R_{dc2} \cos z_2 + x_{s2} R_{ds2} \sin z_2 \\ c_4 &= -x_{s2} R_{dc2} a_2 - \omega_2 x_{s2} R_{ds2} b_2 - \omega_2 x_{s2} R_{dc2} \sin z_2 + \omega_2 x_{s2} R_{ds2} \cos z_2 \\ z_2 &= \frac{2\omega_2 N_1 \pi}{\omega_1} \end{aligned}$$

By solving the simultaneous equation of Eq.(5) and Eq.(7), x_0 , x_1 , v_0 , and v_1 can be obtained, and then the time-history wave of the blade in the steady state can be calculated by substituting x_0 , x_1 , v_0 , and v_1 into Eq.(4) and Eq.(6). Therefore, the maximum value of the time-history wave in the steady state, x_{max} is the maximum response of the blade passing through the asymmetric vane spacing. Reduction factor of the vibratory stress due to asymmetric vane can be defined as Eq.(8).

$$R = 1 - \frac{x_{max}}{(x_{s1}/2\zeta)} \quad (8)$$

This factor means the reduction effect on the vibratory stress with respect to resonant stress for the symmetric vane spacing where the vane count is $2N_1$. Therefore, By using the resonant stress for the symmetric vane spacing calculated by Eq.(1), the maximum vibratory stress for the asymmetric vane, $\sigma_{a,n}$ is calculated by Eq.(9).

$$\sigma_{a,n} = (1-R)\sigma_{s,n} \quad (9)$$

RESULTS OF ANALYSIS AND VERIFICATION TEST

Reduction effect on vibratory stress due to asymmetric vane spacing

In order to examine the reduction effect on vibratory stress due to asymmetric vane, a parametric study on the reduction factor R is carried out by changing the vane count and logarithmic damping of a blade. In this parametric study, the ratio of modal forces in the upper and lower half ($\alpha = F_2/F_1$) is assumed to be unity for simplicity.

Figure 4 shows a typical example of time-history waves of a blade passing through an asymmetric vane spacing calculated

by Eq.(4) and Eq.(6), while Fig. 5 shows the reduction factor R calculated by Eq.(8). As shown in these Figures, by slightly changing the vane count of the upper and lower half, the vibratory stress can be reduced up to 50% of that for the symmetric vane spacing. It can be said that a decrease in vane count and/or logarithmic decrement will increase the amount of response reduction.

Figure 6 shows the reduction factor R , where the abscissa, δN_1 is the product of the logarithmic decrement δ and the vane count of the upper half N_1 . In Fig. 6, the reduction factor corresponding to $N_1 - N_2 = 2$ is the same as that of Fig. 5. From Fig. 5 and Fig. 6, it can be said that the reduction factor R depends on almost only δN_1 if the modal force is not changed in the upper and lower half. And the difference of the reduction factor due to the difference of the vane count is very small. As for a common compressor, the vane count of the upper half is

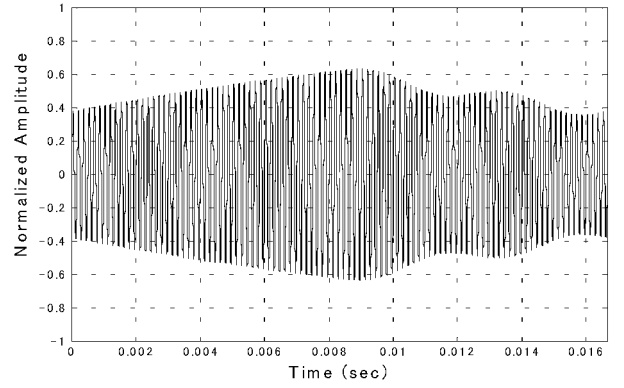


Fig. 4 Resonance response of blade due to asymmetric vane spacing ($N_1=50, N_2=48, \delta=0.01$)

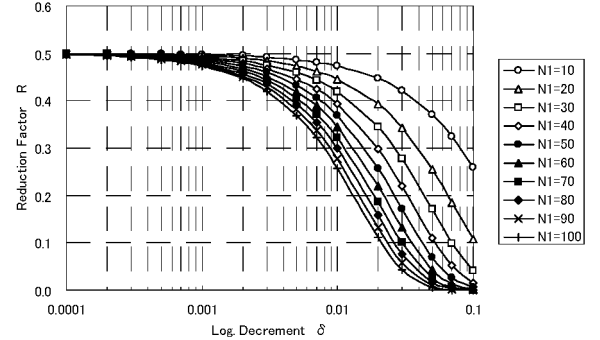
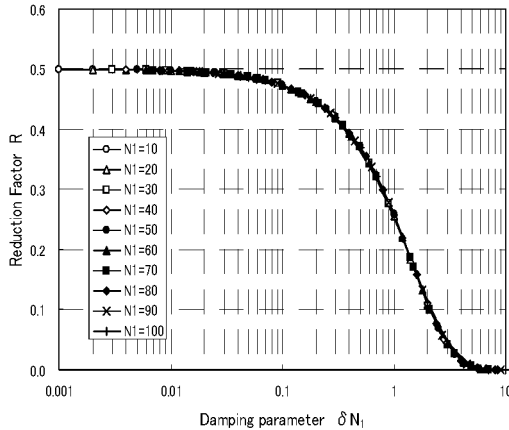
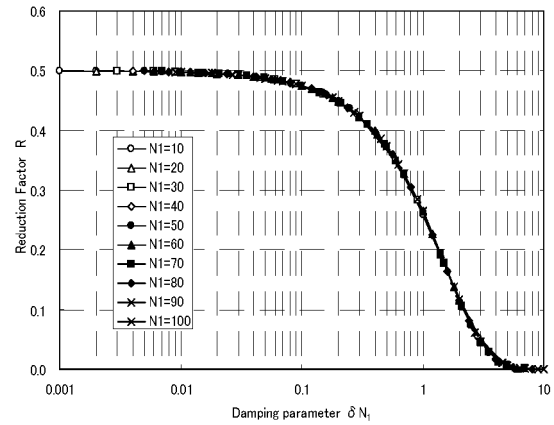


Fig. 5 Reduction factor of asymmetric vane spacing ($N_1 - N_2 = 2, \alpha = 1$)



(a) $N_1 - N_2 = 2$



(b) $N_1 - N_2 = 4$

Fig. 6 Reduction factor of asymmetric vane spacing ($\alpha=1$)

around 20 to 40, and the logarithmic decrement of the higher modes is around 0.01 to 0.02. Therefore, the value of δN_1 can be assumed to be around 0.2 to 0.8. As a result, it is concluded that the reduction effect by 30% to 45% can be expected for the compressor blade by adopting the N=2 asymmetric vane spacing.

Analysis of resonant stress by 3-D CFD and FEM

The resonant stresses of the compressor blade for symmetric and asymmetric vane spacing were calculated according to the theory described in the previous section. Table 1 shows the specification of the blade for analysis, which is the 2nd blade of the compressor test rig consisting of 3 stages and used in the verification test. In the analysis, the resonant stress caused by the interaction force between the 1st vane and the 2nd blade was calculated.

Figure 7 shows the 3-D CFD meshes for symmetric vane spacing, which consists of the 1st vane and the 2nd blade. The ratio of the vane and blade count of 5 to 4 in the computational model is the same as that of the actual compressor. Figure 8 and Fig. 9 show the typical results of CFD analysis. Figure 8 shows the entropy distribution while the Fig. 9 shows the magnitude and phase of the pressure fluctuation on the 2nd blade with blade passing frequency, which was obtained from Fourier analysis of the time-history wave of the pressure fluctuation. Figure 10 shows the vibration mode of the blade calculated by 3-D FEM. This vibration mode is so-called 3-stripe mode or S₁ mode. By using these results, the resonant vibratory stress for the symmetric vane spacing was calculated by Eq.(1). The calculated resonant stress is compared with the measured one, as shown in Fig. 11. Figure 12 shows the whole view of the compressor used in the verification test, and Table 2 shows the test condition. The strain gauges were attached on the blade surface as shown in Fig. 13 and the telemetry system were used to measure the vibratory stress of the blade. In the test, after adjusting the opening of the inlet valve to control the

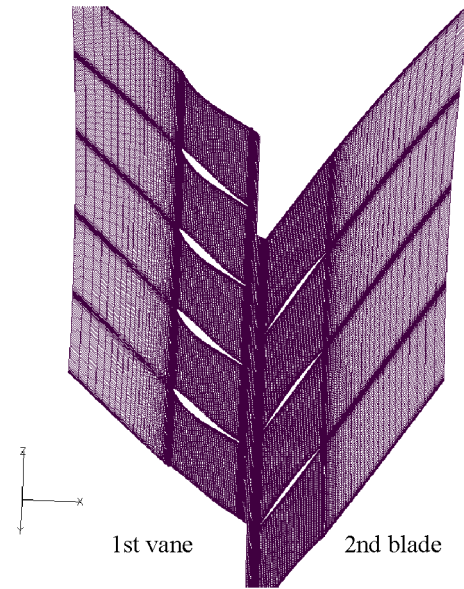


Fig. 7 Example of CFD mesh (50% Ht. section)

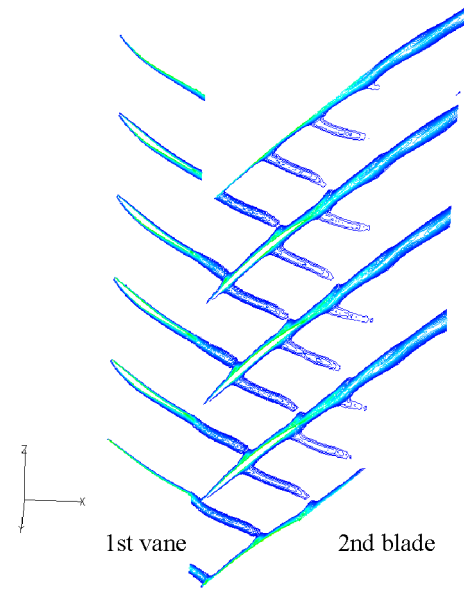


Fig. 8 Example of entropy distribution (50% Ht. section)

Table. 1 Specification of the blade for analysis

Rotation speed	10500 rpm
Blade Height	160 mm
Blade Count	24

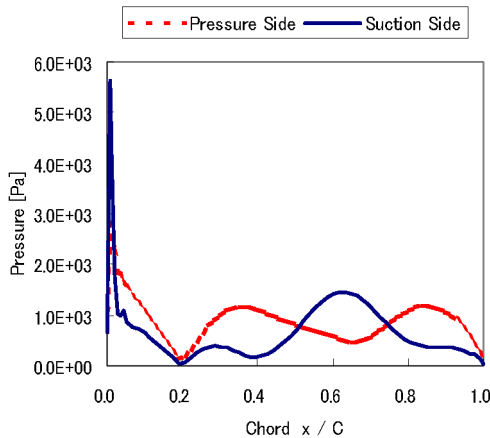
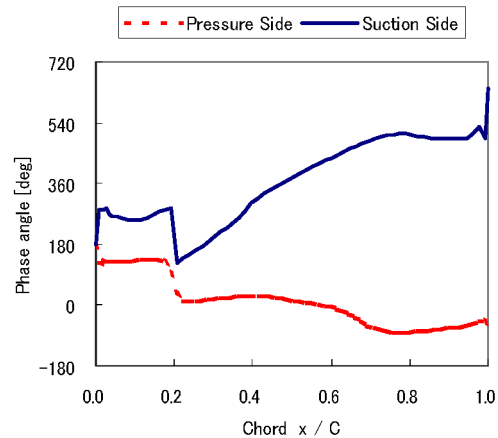


Fig. 9 Magnitude and phase of pressure fluctuation with blade passing frequency (50% Ht. section)



pressure ratio of the compressor, the rotor speed was gradually changed to capture the resonant stress. From Fig. 11, it can be said that the resonant stress of the stripe mode for the symmetric vane spacing can be predicted with the practical accuracy by use of 3-D CFD and 3-D FEM. Figure 14 shows the expected reduction effect of the 2nd blade on the vibratory stress by use of asymmetric spacing vane, which was calculated by Eq.(8). Vane counts of the upper and lower half of the 1st stage used in the analysis is shown in Table 2, which corresponds to the test condition of the verification test for the asymmetric vane spacing described in the next section. The ratio of the modal forces in the upper and lower half ($\alpha=F_2/F_1$) is assumed to be 0.8 to 1.2 based on the results of 3-D CFD analysis. The reduction factor of the 2nd blade on the vibratory

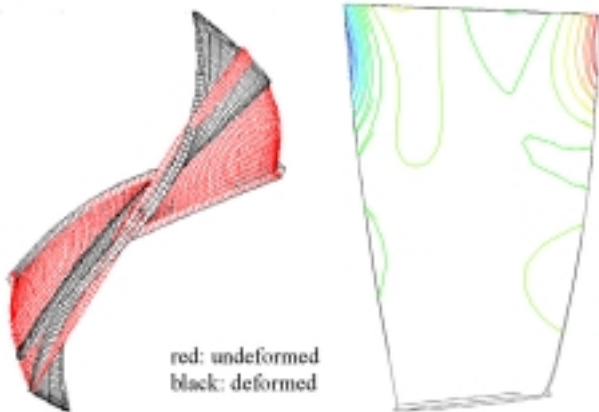


Fig. 10 Vibration mode calculated by FEA (S_1 mode)

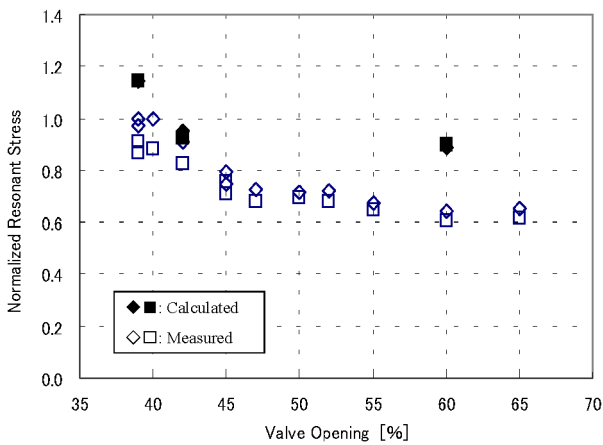


Fig. 11 Calculated and measured resonant stress of the 2nd blade (S_1 mode)

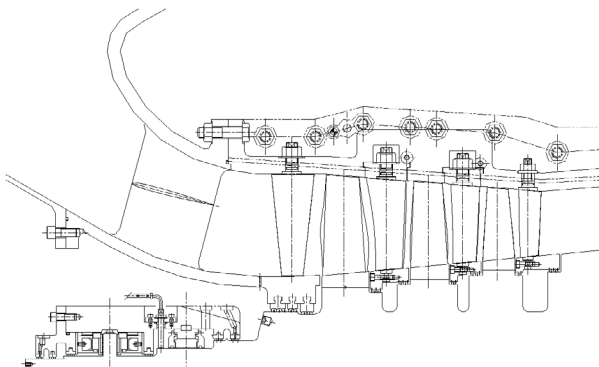


Fig. 12 Whole view of compressor rig

stress due to the asymmetric vane is around 40% to 45% as shown in Fig. 14. The difference of the reduction factor due to the ratio of the modal forces in the upper and lower half is negligible, because the difference of the vane count in the upper and lower half is so small that the modal force in the upper and lower half is not so different. Hence, it is concluded that Fig. 6 can be used in the rough evaluation for reduction effect on the vibratory stress due to the asymmetric vane.

Results of verification test

As a final verification for the reduction effect on the resonant stress due to the asymmetric vane spacing, the compressor rig test was carried out. After finishing the first test for the symmetric vane spacing (Case 1), the casing was disassembled to alter the upper half for the asymmetric vane spacing, and then the test for the asymmetric vane spacing was carried out. In all tests, the operation condition was the same, and the rotor speed was gradually changed so as to measure the completely resonating stress.

Figure 15 shows the resonant stresses of the 2nd blade caused by the interaction force between the 1st vane and the

Table. 2 Vane count of the 1st stage in verification test

Test case		Upper half	Lower half
Case 1	Symmetric	15	15
Case 2	Asymmetric	15	14
Case 3		16	14



Fig. 13 Compressor blade attaching strain gauges

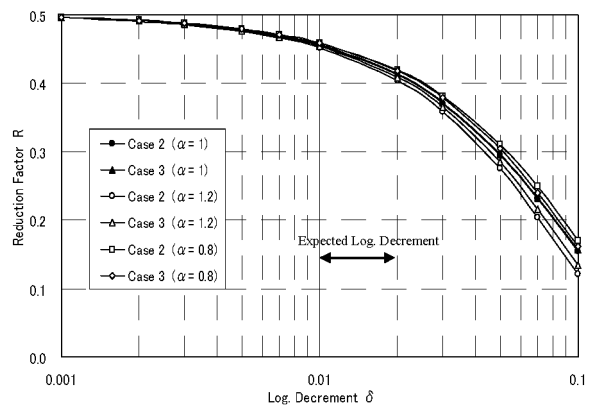


Fig. 14 Expected reduction factor for the 2nd blade

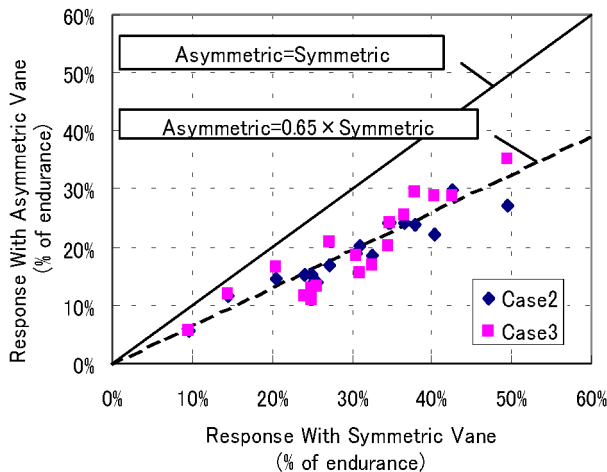


Fig. 15 Resonant stress of the blade caused by blade-vane interaction force

2nd blade. In Fig. 15, the abscissa is the measured resonant stress for the symmetric spacing vane, and the ordinate is for the asymmetric vane spacing. Both resonant stresses are normalized by the fatigue limit of the material and the resonant stresses of all vibration modes resonating around the rated speed are plotted. As shown in Fig. 15, the resonant stresses of all vibration modes for the asymmetric vane are reduced by around 30% to 40%. This reduction effect measured is nearly the same as calculated one. Therefore, it can be said that the reduction effect due to asymmetric vane spacing can be analytically predicted according to the procedure proposed in this paper.

As shown in Fig. 15, the reduction effect on the resonant stress is slightly different for each vibration mode. This reason is probably that the ratio of the modal forces in the upper and lower half is slightly different for each mode as well as the logarithmic decrement.

CONCLUSION

Although Fourier analysis can provide an indication of the response reduction one can expect with asymmetric vanes, it most often overpredicts this reduction. In this paper, an analysis method is proposed for predicting this reduction effect whereby damping and vane count effects are included. The proposed analysis method uses 3-D CFD and modal analysis method based on 3-D FEM. A parametric study was carried out using the proposed method to quantify the effects of vane count and blade damping on the response reduction due to asymmetric vane spacing. This study shows that resonant response reduction approaches 50% with the N=2 vane asymmetry spacing, but that the response reduction is reduced with an increase in vanes and/or damping levels. A rig test with both symmetric and asymmetric vanes validates the proposed analytical method and confirms the reduction effect on vibratory stress due to asymmetric vane spacing.

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