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Recent Findings of Analytical Studies in Unsteady Aerodynamics, Aeroacoustics and Aeroelasticity of Turbomachines

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ABSTRACT

A description is given of the fundamental concept and formulation and solution principles of classical mathematical methods to deal with problems of unsteady aerodynamics, aeroacoustics and aeroelasticity of blade rows. Then recent analytical studies on three model problems are reviewed, and some new findings from the studies are presented. Although the applicability of the analytical methods is under severe restriction, they will keep performing a cruicial role in the preliminary study of new problems.

INTRODUCTION

Recently the share of papers based on analytical methods presented in technical conferences related to aerodynamics and aeroacoustics of turbomachines is becoming smaller and smaller, and CFD (Computational Fluid Dynamics) and CAA (Computational Aeroacoustics) are now playing the dominant role in theoretical studies of turbomachine fluid mechanics. Here analytical methods are meant by 'old-fashioned' theoretical methods to obtain the solution of the governing differential equations in mathematical forms, which explicitly express physical quantities to be calculated in terms of known parameters. The author would like to point out that to obtain the final mathematical forms is not a matter of simple task, and even the analytical methods need to use high speed digital computers with highly complicated computation programs in order to provide final numerical data.

Applicability of most analytical methods is restricted to flows inside and around systems of simple geometries and phenomena where nonlinearity is not essential. On the other hand in principle CFD and CAA can be free from assumptions of small perturbations and linearization, and they can deal with a wide variety of physical systems and physical models if no cost is spared.

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The advantage of analytical methods over CFD and CAA is to enable one to conduct highly quick prediction and extensive parametric studies at a low cost and to gain clear insight into physics. It is true that the advantage is decreasing year by year due to rapid progresses of computer performances and computation algorithms. Further it is also true that the problems which remain to be studied by the analytical methods are becoming scarce.

However CFD and CAA are still too expensive to deal with combined physical models, and there are still some model problems to which analytical methods have been applied but not CFD yet. This paper briefly reviews recent analytical studies conducted by the author:

- Flutter of multiple blade rows.
- Prediction of fan tone noise.
- Active control of gust-rotor interaction noise.

In particular new findings obtained from the analytical studies are highlighted. This review indicates the usefulness of analytical methods for preliminary investigation into key factors and fundamental understanding of new problems. They can also provide benchmarks for code validation of CFD and CAA (Namba and Schulten, 2000). They will keep playing an important role as pilots of advancing the frontiers of knowledge.

SOLUTION PHILOSOPHY OF THE SINGULARITY METHOD

All model problems reviewed herein are analyzed on the basis of the method of singularity. It will be usuful to summarize the common philosophy of solution.

In order for the analytical methods to be applied, it is inevitable to describe the flow field as small perturbations to a uniform steady base flow. Then we assume the small disturbances are convected at the constant velocity of the uniform base flow and propagate at the uniform speed of sound of the base flow. Then the governing equations are linearized, and unsteady phenomena can be described as linear sum of those of harmonic time dependence $e^{i\omega t}$. But it does not necessarily mean each frequency component is independent. In the model problems of multiple blade rows in mutual motion, multiple frequency components are coupled with each other.

Since the base flow is irrotational, acoustic, vortical and entropic disturbances are decoupled in the interior of the fluid, but they are coupled at disturbing solid surfaces, or surfaces of nonzero acoustic admittance.

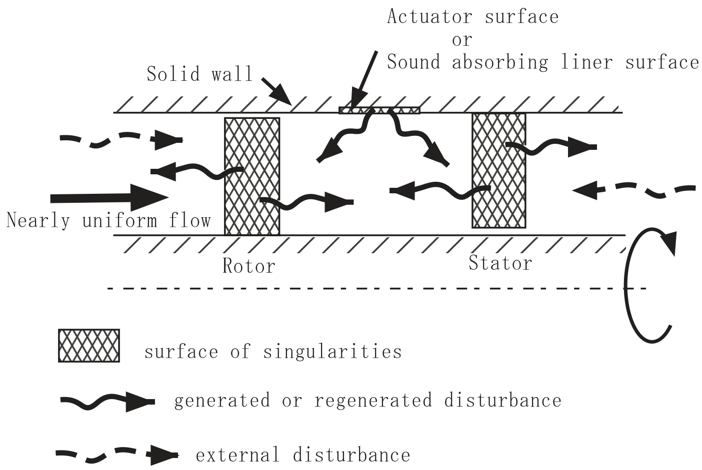

Fig.1: Principles of modeling

All the surfaces which generate or modify the flow disturbances are represented by surfaces of distributed singularities, e.g., monopoles, dipoles, quadrupoles and their combinations. For instance, consider an annular duct model composed of a rotor and a stator and an actuator surface on the duct wall as shown in Figure 1. Then rotor and stator blades which are exerting unsteady force upon the fluid are represented as surfaces of unsteady pressure dipoles of axses normal to the surfaces, while the actuator surface is expressed by a surface of unsteady mass sources (monopoles combined with dipoles with streamwise axses. Furthermore, if blades are vibrating under nonzero steady loadings, then we have additional singularities of various types (dipoles of streamwise axses, quadrupoles, etc.) with strength proportional to steady force times displacement amplitude.

Let $\boldsymbol{q}(\boldsymbol{x})$ denote the column vector of the disturbance state variables (complex amplitude to be multiplied by $e^{i\omega t}$); density ρ, velocities u, v, w and pressure p, i.e.,

$$\boldsymbol{q}(\boldsymbol{x}) = (\rho, u, v, w, p)^{\mathrm{T}}. \tag{1}$$

Here $\boldsymbol{x}$ denotes the position vector at a field point. Further let the strength of singularities at a point $\boldsymbol{\xi}$ be denoted by

$$\boldsymbol{f}(\boldsymbol{\xi}) = (f_1, f_2, \ldots, f_n)^{\mathrm{T}}, \tag{2}$$

where subscripts identify singularity surfaces and types of singularities. Then the disturbances are expressed as a sum of all disturbances generated from the singularity surfaces in the form:

$$\boldsymbol{q} = \mathbf{K} * \boldsymbol{f}, \tag{3}$$

where $\mathbf{K}(\boldsymbol{x}-\boldsymbol{\xi})$ denotes a matrix of kernel functions:

$$\mathbf{K}(\boldsymbol{x}-\boldsymbol{\xi}) = \begin{pmatrix} K_{\rho 1} & \cdots & K_{\rho n} \\ K_{u1} & \cdots & K_{un} \\ \vdots & \ddots & \vdots \\ K_{p1} & \cdots & K_{pn} \end{pmatrix}. \tag{4}$$

Here for instance, $K_{uk}(\boldsymbol{x}-\boldsymbol{\xi})$ denotes disturbance $u(\boldsymbol{x})$ induced by the k-th singularity of unit strength at $\boldsymbol{\xi}$.

In general each kernel function is a solution to an inhomogeneous Helmholtz equation in the form like

$$D_H K_{uk}(\boldsymbol{x}-\boldsymbol{\xi}) = D_{uk}\delta(\boldsymbol{x}-\boldsymbol{\xi}), \tag{5}$$

satisfying the out-going wave conditions and the flow tangency condition at the solid duct walls. Here D_H denotes the differential operator of the Helmholtz equation and D_{uk} denotes an integro-differential operator appropriate for the type of the singularity. Furthermore $*$ denotes the convolution product over the domain of the singularity surface.

The strength of the singularities are determined so that they satisfy the boundary conditions at singularity surfaces, which can be written in the form

$$\mathbf{A}\,[\mathbf{K}]_{\mathrm{BS}} * \boldsymbol{f} = \boldsymbol{b}, \tag{6}$$

where

$$\boldsymbol{b}(\boldsymbol{x}) = (b_1, b_2, \ldots, b_n)^{\mathrm{T}}, \tag{7}$$

denotes prescribed external disturbances at the singularity surfaces, e.g., downwash velocity of incoming acoustic disturbances or incoming vortical disturbances or velocity of blades relative to the fluid in the case of blade vibration. Furthermore $\mathbf{A}$ denotes an appropriate coefficient matrix which can be specified in accordance with the surface position, the types of singularities and the types of the external disturbances.

What we should do is to solve differential equation (5) for each kernel function, and to solve equation (6) for singularity strength $\boldsymbol{f}$, which usually take a form of simultaneous integral equations. Then we can evaluate the blade loadings from $\boldsymbol{f}$ itself, and the disturbances $\boldsymbol{q}(\boldsymbol{x})$ at an arbitrary field point by calculating the convolution product given by equation (3).

One of the major tasks of the analytical methods is to express the kernal functions in 'closed' forms as solutions to the inhomogeneous Helmholtz equations like (5). This is not always easy, and demands sophisticated mathematical skills. On the other hand it is quite rare to be able to obtain an analytical solution of a set of integral equations (6), and usually they should be solved numerically. For this reason the methods are often called 'semi-analytical methods'. It is not unusual that the expressions of kernel functions for the problems of three-dimensional unsteady flows are highly complicated. Therefore use of high speed digital computers and careful coding of computation programs are indispensable for conducting the numerical tasks based on the analytical methods, too.

The present formulation is somewhat different from that used by Topol (Topol, 1997), Hanson (Hanson, 1997) and Hall and Silkowski (Hall *et al.*, 1997). In their papers explicit discrimination is made between disturbances going upstream and downstream, and also between acoustic and vortical disturbances. In addition mechanism of reflection and transmission of disturbances due to interaction with blade rows are also explicitly described.

In our formulation the kernel functions satisfy the outgoing wave condition and automatically include acoustic waves going upstream and downstream as well as vortical and entropic disturbances being convected downstream. Equation (3) takes a form of a linear superposition of disturbances generated from all singularity surfaces as if they are individually independent disturbance sources. But it in perfectly proper way describes the disturbance flow field induced by original disturbances $\boldsymbol{b}$ and involving reflection and transmission, because of determination of the singularitiy strengths $\boldsymbol{f}$ from the boundary condition (6) through which all singularities are coupled with each other.

FLUTTER OF MULTIPLE BLADE ROWS

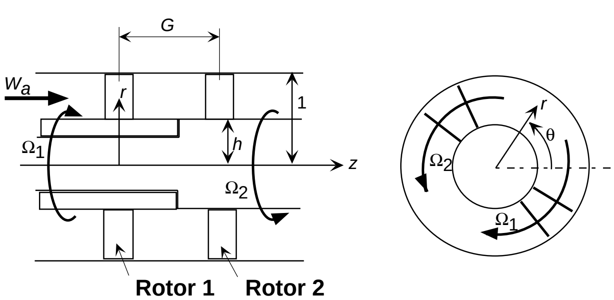

Fig.2: Contra-rotating annular cascades.

To compute the unsteady flow field of rotor-stator interaction is one of the main CFD problems of turbomachines, and many successful results have been obtained so far. However, to deal with flutter problems of multiple annular cascades in mutual motion will still be an extremely awkward task for CFD. But application of the linearized cascade theory or lifting surface theory for cascades to this problem is essentially of no difficulty.

With respect to the flutter problem of multiple blade rows, special mention must be made of pioneering works conducted in Japan about thirty years ago. The theoretical study based on the semi-actuator disk model by Tanida (Tanida, 1966) and the experiments and the linear cascade theory by Kobayashi et al (Kobayashi *et al.*, 1974, Kobayashi *et al.*, 1975) should be cited as the earliest works. Their theories assume incompressible flows and therefore are unable to account for aeroacoustic interaction between blade rows via cut-on acoustic duct modes. About fifteen years later Butenko and Osipov (Butenko *et al.*, 1989) developed a theory for oscillating subsonic linear cascades in relative motion.

The subject is recently attracting a new attention of turbomachine aerodynamicists. Hall and Silkowski (Hall *et al.*, 1997) presented an analysis based on a linearized model of two-dimensional subsonic multiple blade rows.

The author's research group has conducted analytical studies on the contra-rotating annular cascades as shown in Figure 2, where blades of one of the rotors are vibrating under mutual aerodynamic interaction between both rotors. There have been developed three-dimensional lifting surface theories for combinations of subsonic and subsonic rotors (Namba *et al.*, 2001), supersonic and supersonic rotors (Namba, 2001), and subsonic and supersonic rotors (Namba *et al.*, 2003).

Those studies indicate that the influence of aeroacoustic coupling among blade rows on the aerodynamic damping force is closely related to the states (cut-off, cut-on or near resonance) of the acoustic duct modes generated from oscillation of blades and aeroacoustic interaction of both rotors in mutual motion.

The distinguishing feature of the problem is multiplication of frequency and circumferential modes due to interaction of blade rows in mutual motion.

Let numbers of blades of rotor 1 and rotor 2 be N_{B1} and N_{B2} respectively, and rotational angular velocities of rotor 1 and rotor 2 be Ω_1 and Ω_2 respectively. Assume that the blades of rotor 1 are vibrating with a single frequency ω_{10} and an inter-blade phase angle $2\pi\sigma_{10}/N_{B1}$, so that the displacement normal to the blade chord of the m-th blade is given by

$$a_1(r,z)e^{i\omega_{10}t+i2\pi\sigma_{10}m/N_{B1}} \; : \; m=0,1,...,N_{B1}-1. \quad (8)$$

Here σ_{10} is an integer between $-N_{B1}/2$ and $N_{B1}/2$.

Then as described in details in references (Namba *et al.*, 2001, Namba, 2001), aeroacoustic coupling between the rotors in mutual motion produces flow disturbances of multiple frequencies, resulting in blade loading of multiple frequencies. Thus we must describe the unsteady blade loading (pressure difference between upper and lower surfaces of blades) on the m-th blade of each rotor as summations of multiple frequency components:

$$\sum_{\nu=-\infty}^{\infty} \Delta p_{1-1,(\nu)}(r,z_1)e^{i\omega_{1\nu}t+i2\pi\sigma_{1\nu}m/N_{B1}}$$

and

$$\sum_{\mu=-\infty}^{\infty} \Delta p_{2-1,(\mu)}(r,z_2)e^{i\omega_{2\mu}t+i2\pi\sigma_{2\mu}m/N_{B2}}, \quad (9)$$

where

$$\begin{aligned} \omega_{1\nu} &= \omega_{10} - \nu N_{B2}(\Omega_1 - \Omega_2), \\ \sigma_{1\nu} &= \nu N_{B2} + \sigma_{10}, && (10) \\ \omega_{2\mu} &= \omega_{10} + (\mu N_{B1} + \sigma_{10})(\Omega_1 - \Omega_2), \\ \sigma_{2\mu} &= \mu N_{B1} + \sigma_{10}. && (11) \end{aligned}$$

Further subscripts 1-1 and 2-1 imply the loading on rotor 1 blades due to vibration of rotor 1 blades themselves and the loading on rotor 2 blades due to vibration of rotor 1 blades respectively. It is worth emphasizing that all frequency components are coupled with each other and can not be determined independently. It implies that $\boldsymbol{f}$ in equation (6) is composed of multiple frequency components, for which a set of equations should be solved simultaneously.

In the present problem acoustic disturbances are also composed of multiple frequency components and multiple acoustic duct modes. The frequencies $\omega_{\nu,\mu}$ viewed in the frame of reference fixed to the duct, and corresponding circumferential wave numbers $n_{\mu,\nu}$ of the duct modes are given by

$$\begin{aligned} \omega_{\nu,\mu} &= \omega_{10} + \mu N_{B1}\Omega_1 + \nu N_{B2}\Omega_2 + \sigma_{10}\Omega_1, \quad (12) \\ n_{\mu,\nu} &= \mu N_{B1} + \nu N_{B2} + \sigma_{10}. \quad (13) \\ (\mu,\ \nu &= 0, \pm 1, \pm 2, \ldots) \end{aligned}$$

respectively. Hereafter we denote the acoustic duct mode of $(n_{\mu,\nu}, \ell)$ by $(\mu, \nu;\ \ell)$, where ℓ denotes the radial order.

Under this notation we can state that vibrating blades directly generate $(\mu, 0;\ \ell)$ modes, which we call primary duct modes. On the other hand the duct modes with $\nu \neq 0$ are those arising from blade row interaction and let those be called secondary duct modes. If all of the primary modes are cut-off and if the rotors are remotely separated, the influence of the neighboring blade row will not be substantial.

We should note, however, that there also exist vortical disturbances which are convected without decaying. Therefore in the case of vibration of rotor 1, the vortical disturbances from rotor 1 always exert a finite influence on rotor 2 even if all primary duct modes $(\mu, 0;\ \ell)$ are cut-off and however large the rotor-to-rotor distance G may be. Further, any of the modes of $\nu \neq 0$ resulting from the interaction can be cut-on, giving backward reaction to rotor 1. The previous studies (Namba *et al.*, 2001) indicate, however, that the vortical disturbances play only a minor role in the aerodynamic interaction between the blade rows.

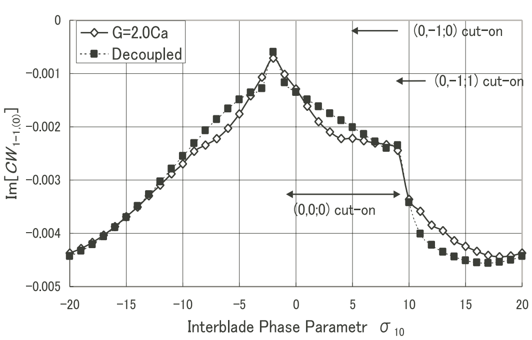

Fig.3: The work coefficient $\Im[CW_{1-1,(0)}]$ for bending vibration of rotor 1. Axial Mach number $M_a = 0.6$, boss ratio $h = 0.7$, number of blades: $N_{B1} = N_{B2} = 40$, axial chord/duct radius: $C_{a1} = C_{a2} = 0.1$, blade tip speed/axial velocity: $\Omega_1 = -\Omega_2 = 1.0$, reduced frequency: $\omega_{10}C_{a1} = 0.5$. Case of subsonic rotor 1 and subsonic rotor 2

As a measure of unsteady blade loading a generalized force coefficient $CW_{j-k,(\mu)}$ is defined by

$$CW_{j-k,(\mu)} = \pi \int_h^1 \sqrt{1 + \Omega_j^2 r^2}\, dr$$

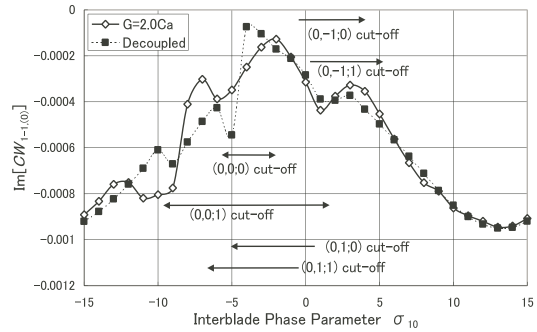

Fig.4: The legend is same as Fig.3, except; number of blades: $N_{B1} = 30$, $N_{B2} = 40$, axial chord/duct radius: $C_{a1} = 0.0633$, $C_{a2} = 0.1$, blade tip speed/axial velocity: $\Omega_1 = 3.0$, $\Omega_2 = -1.0$, reduced frequency $\omega_{10}C_{a1} = 0.5$. Case of supersonic rotor 1 and subsonic rotor 2

$$\times \int_{C_{aj}/2}^{C_{aj}/2} \Delta p_{j-k,(\mu)}(r, z_j)\overline{a_j(r, z_j)}\, dz_j, \quad (14)$$

where an overlined symbol denotes the complex conjugate. Then the aerodynamic work per cycle on the vibrating blades of rotor 1 is given by $\Im[CW_{1-1,(0)}]$

Example calculations are shown in Figures 3 and 4, where the distance between the rotor centers is $G = 2.0(C_{a1} + C_{a2})/2$. Results of the isolated blade row (Decoupled) are also plotted for comparison. Deviation from the values for the isolated blade row (Decoupled) is regarded as the influence of the neighboring blade row.

In the case of vibrating subsonic cascade (Fig. 3), only a small number of primary duct modes are cut-on. In the range of interblade phase angle where the fundamental primary duct mode (0,0;0) is cut-on the influence of the neighbor cascade is not small and it never decreases with increase of rotor-to-rotor distance G. Although we can also observe some influence of neighbor cascade in the range where (0,0;0) mode is cut-off, the influence in this range is decreased by increasing G.

In the case of vibrating supersonic cascade (Fig. 4), infinite number of primary duct modes are cut-on. In this case conspicuous influence of neighbor cascade is observed around resonance points of duct modes of low orders.

A computation program to solve the coupled bending-torsion flutter equations for contra-rotating annular cascades was also formulated and coded.

Note that to search the critical flutter condition, we should compute the aerodynamic force terms for various values of the reduced frequency of blade vibration. To this end computation by CFD may be too time-consuming. On the other hand the present analytical method is a very useful aerodynamic tool, which can provide numerical values of aerodynamic force terms for a given reduced frequency within a few seconds on a conventional personal computer.

In the present flutter analysis the elastic properties of blades are assumed uniform along the span and natural mode shapes are assumed same as those for the uniform flat plate. Vibration modes of the first and second bending orders with the natural frequencies ω_{B1} and ω_{B2} and the first and second torsion orders with the natural frequencies ω_{T1} and ω_{T2} are taken into account. The specified values are as follows; the mass ratio $M_b/(\pi\rho_0 b_a^2 r_T(1-h)) = 120$, the normalized distance between the center of gravity and the elastic axis $x_{eg}/b_a = 0.141$, the normalized radius of gyration $r_e/b_a = \sqrt{0.6}$, the elastic axis position $z_e/C_a = -0.075$, and the natural frequency ratios $\omega_{T1}/\omega_{B1} = 6.0$, $\omega_{B2}/\omega_{B1} = 6.3$, $\omega_{T2}/\omega_{B1} = 18.0$. Here M_b is the mass of a blade, and $b_a = C_a/2$.

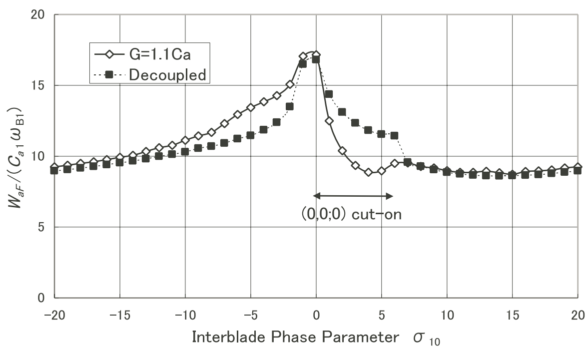

Fig.5: Critical axial flow velocity for coupled bending-torsion flutter in case of combination of subsonic and subsonic rotors.

Figure 5 shows the dimensionless critical axial flow velocity at the flutter boundary dependent on interblade phase parameter σ_{10} . The aerodynamic and geometrical conditions are same as those of Figure 3 except the blade row distance $G = 1.1C_a$. We can observe that the flutter velocity of the coupled blade rows is significantly lower than that of the decoupled blade row in the region where the duct mode $(0, 0; 0)$ is cut-on.

The results of the studies suggest that the flutter characteristics will be substantially modified by the presence of neighbor cascades.

PREDICTION OF FAN TONE NOISE

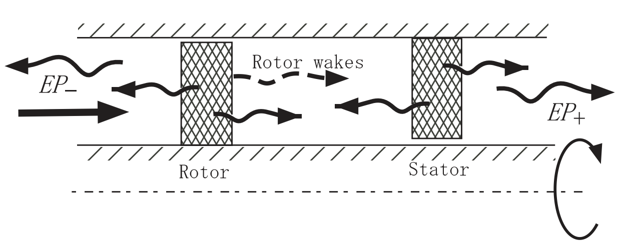

Fig.6: Model for predicting a coupled fan noise.

It is a common understanding that interaction of stator vanes with oncoming rotor wakes is the main source of the fan noise of turbofan engines. In earlier models the upstream rotor is treated only as a wake generator and interaction of the rotor blades with sound waves generated from the stator was neglected. But recent models (Topol, 1997) (Hanson, 1997) include the effects of mutual aeroacoustic interaction between the rotor and the stator, and it is shown that the scattering effects are substantial. But their models compute the scattering effects on the basis of 2-dimensional cascade theory.

A prediction scheme on the basis of genuine three-dimensional lifting surface theory has been developed by the author for the model shown in Figure 6. Mathematical formulations can be achieved with a minor modification of those for the previous section, i.e., we can just put the rotational speed of the downstream rotor to zero and express $\boldsymbol{b}$ in equation (6) by the wake upwash velocity on the stator vane instead of relative upwash due to blade vibration.

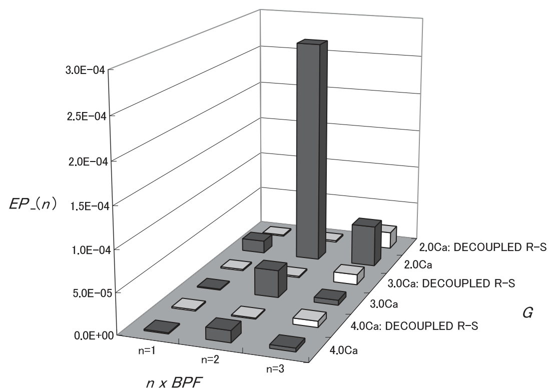

Fig.7: Acoustic powers of fan noise caused by rotor wakes interacting with the stator vs the distance G between the rotor and the stator. 'Decoupled R-S' means no aeroacoustic interaction between the rotor and the stator. Boss ratio $h = 0.5$, rotor tip speed /axial flow velocity $\Omega_1 = 1.566$, axial chord/duct radius $C_{a1} = 0.1409$ (rotor) and $C_{a2} = 0.2618$ (stator), number of the rotor blades $N_{B1} = 24$, number of the stator vanes $N_{B2} = 32$, axial Mach number $M_a = 0.5$. Upstream acoustic power EP_-.

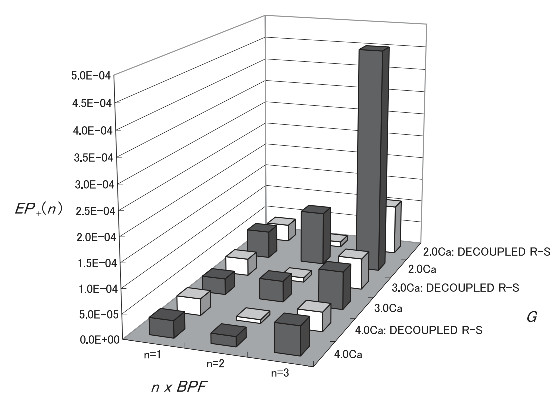

Fig.8: The legend is same as Fig.7, except: Downstream acoustic power EP_+.

Figures 7 and 8 show example calculations of acoustic powers for the fundamental blade passing frequency, the second and the third harmonic frequencies. In this case the component of the fundamental blade passing frequency is not predominant, because duct modes for the fundamental blade passing frequency are close to cut-off.

It is obvious that the acoustic power decreases as the distance G between the rotor and the stator increases. Special attention should be payed to the substantial difference between the decoupled case and the coupled case. In particular dependence on the distance G in the coupled case is much higher than that in the decoupled case. This implies that disturbances of cut-off modes play a significant role in near field coupling. It indicates that to take into account the aeroacoustic coupling between the rotor and the stator is essentially important for precise prediction of fan tone noise.

Recently Tsuchiya et al. (Tsuchiya *et al.*, 2002) proposed a hybrid scheme, which calculates the unsteady blade loading due to interaction with oncoming rotor wakes by CFD and predicts the acoustic field by the lifting surface theory. In order to make clear the level of the approximation, let the blade loading function $\boldsymbol{f}$ be expressed as

$$\boldsymbol{f} = \boldsymbol{f}_w + \boldsymbol{f}_c, \tag{15}$$

where $\boldsymbol{f}_w$ denotes the blade loading due to interaction with rotor wakes and without the rotor-stator aeroacoustic coupling, while $\boldsymbol{f}_c$ denotes the additional blade loading due to the rotor-stator aeroacoustic coupling. Then equations (1) and (6) are written as

$$\boldsymbol{q} = \boldsymbol{K}*(\boldsymbol{f}_w + \boldsymbol{f}_c), \tag{16}$$

$$\mathbf{A}[\mathbf{K}]_{\mathrm{BS}}*\boldsymbol{f}_c = \boldsymbol{b} - \mathbf{A}[\mathbf{K}]_{\mathrm{BS}}*\boldsymbol{f}_w. \tag{17}$$

On the other hand the boundary condition under formulation without the aeroacoustic coupling between the rotor and stator can be written as

$$\mathbf{A}[\mathbf{K}_{dc}]_{\mathrm{BS}}*\boldsymbol{f}_w = \boldsymbol{b},, \tag{18}$$

where the terms denoting the rotor-stator coupling are absent in the kernel function matrix $\mathbf{K}_{dc}$.

It is the author's understanding that Tsuchiya et al.'s scheme is to numerically solve the original differential equations by CFD to obtain $\boldsymbol{f}_w$ instead of solving linearized equation (18), and to compute the acoustic field by

$$\boldsymbol{q} = \mathbf{K}*\boldsymbol{f}_w. \tag{19}$$

To compute $\boldsymbol{f}_w$ by CFD is certainly an excellent idea. The scheme may be, however, incomplete because the aeroacoustic coupling effect $\boldsymbol{f}_c$ is neglected. It will be much more improved by solving equation (17) for $\boldsymbol{f}_c$ with substituting $\boldsymbol{f}_w$ computed by CFD, and predict the acoustic field by equation (16).

ACTIVE CONTROL OF GUST-ROTOR INTRACTION NOISE

Application of the anti-sound technology to suppression of tone noise due to interaction of blade rows with oncoming wakes or gusts is quite attractive because of its high adaptability. This idea is not new, and some studies have been conducted. However, experiments by Thomas et al. (Thomas *et al.*, 1994) and Ishii et al. (Ishii *et al.*, 1997) were not necessarily successful in case of multiple cut-on modes.

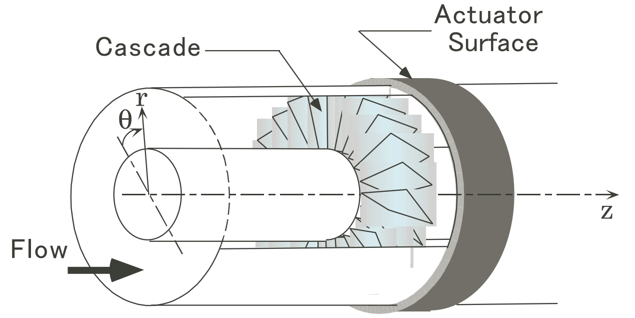

Fig.9: Annular cascade with an actuator surface on the duct wall.

Recently analytical studies have been conducted on the model shown in Figure 9 (Namba and Murahashi, 1999) (Namba *et al.*, 1999). The rotor with a number of blades N_B and a rotational speed Ω is supposed to interact with oncoming sinusoidal gust of a circumferential wave number N_G which is stationary in the duct frame of reference. As an anti-sound generator a duct wall actuator surface like a loudspeaker diaphragm is considered.

The gust-rotor interaction induces an unsteady blade loading with a frequency $N_G\Omega$ and an interblade phase angle $2\pi N_G/N_B$. It generates tone noise of the blade passing frequency and its harmonics $\nu N_B \Omega$ and circumferential wave numbers $\nu N_B + N_G$: $(\nu = \pm 1, \pm 2, \ldots)$. The duct modes are spinning at angular velocities of $\nu N_B\Omega/(\nu N_B + N_G)$, and they are further decomposed into multiple radial modes of mode order $\ell = 0, 1, \ldots$.

The actuator surface is made oscillate in a circumferentially travelling wave form at a frequency $\nu^* N_B \Omega$ and with a circumferential wave number of $\nu^* N_B + N_G$. This motion generates duct modes of the circumferential wave number of $\nu^* N_B + N_G$ and the spinning angular velocity of $\nu^* N_B\Omega/(\nu^* N_B + N_G)$ and also multiple radial orders.

Here we should note that interaction of the disturbances generated from the actuator with the rotor blades induces additional unsteady blade loading with the same frequency and the same interblade phase angle as those induced by the gust-rotor interaction, and generates additional tone noise with the same frequencies and the same circumferential wave numbers as those of gust-rotor interaction noise.

The blade loadings and the modal pressure amplitudes can be calculated by use of the analytical method. Denoting the mode order as (ν, ℓ), we can express the modal pressure amplitude which is a complex number in the form

$$\begin{aligned} FP_{\pm}(\nu,\ell) &= Q_G FP_{BG\pm}(\nu,\ell) \\ &+ B_{\nu^*}\{FP_{W\pm}(\nu^*,\ell)\delta_{\nu,\nu^*} + FP_{BW\pm}(\nu,\ell)\}. \end{aligned} \tag{20}$$

Here subscripts BG, W and BW denote components of the gust-rotor interaction, the direct sound from the actuator and the actuator-rotor interaction respectively and $+$ and $-$ denote acoustic waves propagating in downstream and upstream directions respectively. Furthermore Q_G denotes the gust velocity amplitude normalized by the axial flow velocity, and B_{ν^*} denotes the actuator displacement amplitude normalized by the duct radius. The acoustic power is expressed in the form

$$EP_{\pm} = \sum_{\nu} \sum_{\ell} S_{\ell\pm}^{(\nu)} |FP_{\pm}(\nu, \ell)|^2, \tag{21}$$

where $S_{\ell\pm}^{(\nu)}$ is a coeffcient and summations are made only for cut-on modes.

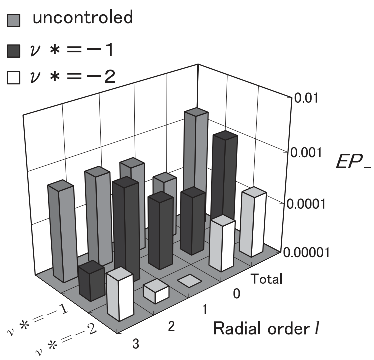

Fig.10: Dependence of the minimized upstream acoustic power EP_- and its modal components on the actuator exciting mode ν^*.

The main problem is to determine the complex amplitude of the actuator B_{ν^*} to minimize $EP_{\pm}$ and also to find the best choice of the exciting mode ν^* of the actuator. The analytical method enables one to obtain a mathematical expression of the optimum amplitude B_{ν^*} in terms of design parameters.

Figure 10 shows an example of the minimized acoustic power dependent on the exciting mode ν^* of the actuator. In this case only the duct modes of $\nu = -1$, $\ell = 0, 1, 2, 3$ are cut-on. It is a surprising finding that excitation of the actuator with the cut-on circumferential wave number $\nu^* = -1$ is unsuccessful, and the cut-off mode number $\nu^* = -2$ is the best choice.

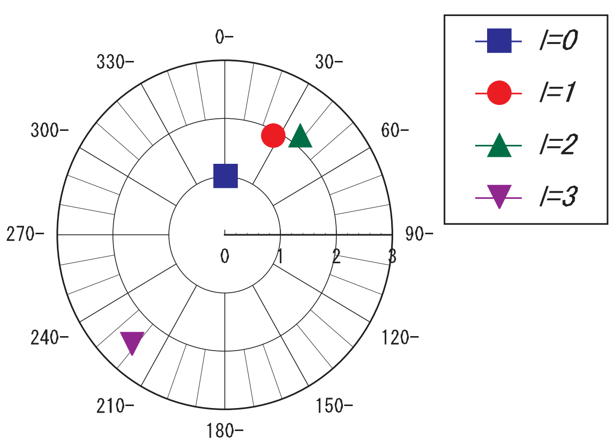

Fig.11: Modal structure of the component $FP_{BG-}(-1, \ell)$.

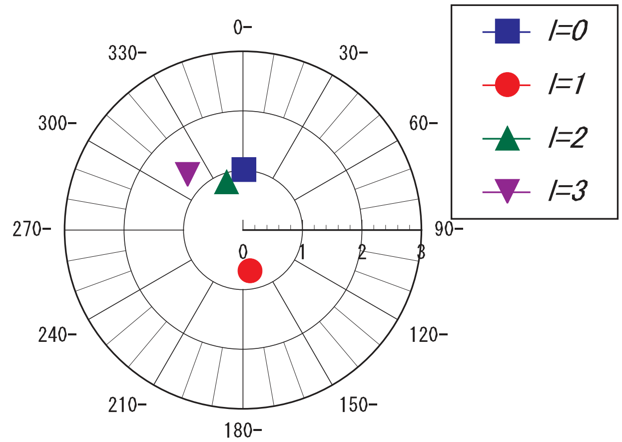

Fig.12: Modal structure of the component $FP_{W-}(-1, \ell)$ for $\nu^* = -1$.

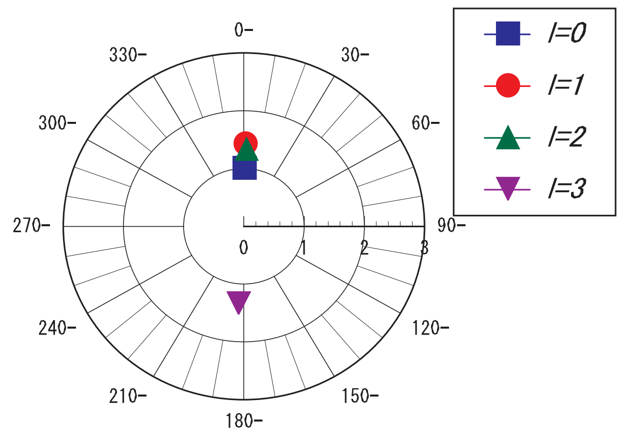

Fig.13: Modal structure of the component $FP_{BW-}(-1, \ell)$ for $\nu^* = -1$.

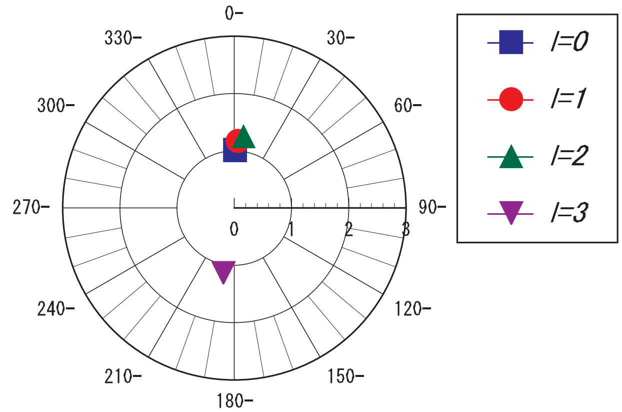

Fig.14: Modal structure of the component $FP_{BW-}(-1, \ell)$ for $\nu^* = -2$.

Closer investigation into the modal structures of each component of the pressure amplitude shown in Figures 11 - 14 reveals that relative relationship of complex amplitudes among different radial orders of $FP_{W\pm}(\nu, \ell)$ is essentially different from that of $FP_{BG\pm}(\nu, \ell)$, while $FP_{BW\pm}(\nu, \ell)$ and $FP_{BG\pm}(\nu.\ell)$ have similar relationship. Therefore simultaneous cancellation of all cut-on modes of $FP_{BG\pm}(\nu, \ell)$ by $FP_{W\pm}(\nu, \ell)$ themselves is impossible.

Instead, we can adopt a strategy of letting the actuator motion iteself generate $FP_{W\pm}(\nu^*,\ell)$ of cut-off modes only, and canceling $FP_{BG\pm}(\nu,\ell)$ by the 'secondary' noise $FP_{BW\pm}(\nu,\ell)$. This deserves to be called the aeroacoustic control rather than the acoustic control. The author would like to emphasize that such a finding can only be made by an extensive study on the basis of genuine three-dimensional cascade models.

CONCLUSIONS

- The flutter boundary of cascading blades is significantly influenced by the presence of neighbor cascades in mutual motion.
- The interaction of fan blade wakes with stator vanes is considered as the main source of fan noise of turbofan engines. But to accurately predict the sound field, the aeroacoustic coupling between fan blades and stator vanes should be taken into account.
- Direct cancellation of gust-rotor interaction noise of multiple cut-on modes by the anti-sound from the secondary sound sources on the duct wall is in general unsuccessful, because the modal structures of the noise and the anti-sound are essentially different. Instead the noise may effectively be reduced by generating secoundary sound composed of cut-off modes only, and a new concept of aeroacoustic control of noise arises.

Analytical methods will keep the usefulness for preliminary study of frontier problems and fundamental understanding of physics.

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