The challenges of lean premixed combustion

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ABSTRACT

Clean combustion, with low levels of pollutants, can be achieved by mixing the fuel and air well before they enter a combustion chamber and burning lean with a high ratio of air to fuel. Unfortunately, these are just the conditions that cause self-excited oscillations which can become so intense that they break the combustor. Many gas-fuelled power stations have suffered damage in this way and been unable to achieve their designed power and low emission levels. Self-excited oscillations are also being experienced in the development of the next generation of aeroengines as they strive to achieve lower emissions. These instabilities occur due to the interaction between sound and flames: the unsteady combustion generates acoustic waves and these in turn alter the inlet flow rates of fuel and air and lead to further unsteady combustion.

An overview of the physics of the interaction between sound and flames is presented and will be illustrated by visualisations of flame dynamics through experiment and computer modelling. The elements needed for effective prediction and modelling are discussed, together with the potential of 'anti-sound' and passive acoustic absorbers to eliminate the instability.

INTRODUCTION

Gas turbine combustors which burn lean and premixed lead to very low levels of nitrous oxides. Manufacturers of industrial gas turbines, which have to meet stringent emission requirements, have therefore vigorously pursued lean premixed (LP) technology. Unfortunately LP combustion has proved to be notoriously susceptible to instability leading to large-amplitude pressure oscillations. These may cause structural damage, either because the resulting flow perturbations enhance the heat transfer or through vibration. More than one power station has been unable to achieve its designed power and low emissions because of the onset of instability, leading to long commissioning times as a series of ad hoc fixes are tried. Lean premixed combustion is also of interest to aeroengine manufacturers since the next generation of aero gas turbines is also required to achieve very low emissions. Self-excited oscillations are being experienced in their development.

Historically, self-excited combustion instabilities have been experienced in combustors with a high specific heat input like afterburners and rockets, and occur as the fuel-air ratio is increased above a critical level. Lean premixed combustion is different, with intense oscillations at low fuel-air ratios. The physics of this mechanism are now clear: pressure oscillations cause changes in the flow rates of inlet air or fuel, thereby changing the fuel-air ratio. At lean premixed conditions, the rate of combustion is strongly influenced by fuel-air ratio, indeed if the fuel-air ratio drops significantly local extinctions can occur. The resulting change in the rate of combustion is an acoustic source, generating pressure waves. Instability is then possible because while the unsteady combustion generates sound, the resulting pressure waves cause yet more unsteady combustion. If the phase relationship is suitable, linear perturbations grow in amplitude into a finite amplitude self-excited oscillation.

In this paper, we present an overview of the physics of the interaction between sound and flames. Methods to predict the susceptibility to self-excited oscillation and the frequency of these oscillations are presented. The complementary roles of experiments, wave analysis, Computational Fluid Dynamics and low-order models for unsteady combustion are discussed. These approaches have been validated by experiments on simple lean premixed flames and can be applied to predict oscillations in gas turbine combustors.

Many LP combustion systems are unstable and good design will include passive or active control features. Passive control usually takes the form of either a redesign of the premix passages and combustor to reduce the sensitivity of the combustion to changes in the inlet flow, or the introduction of additional acoustic absorbers. Active control, in which the fuel is added unsteadily in response to a measured fluctuating signal, is an effective way of stabilising combustion oscillations. It is over ten years since the first full-scale demonstration of the feasibility of such feedback control. However, practical implementation requires controllers that have guaranteed performance across a range of combustor operating conditions. The development of such controllers needs the integration of ideas from acoustics, fluid mechanics, combustion, control and system identification. Current work on adaptive controllers will be summarised, with particular emphasis on ensuring that the controller will operate effectively as the engine conditions change.

INTERACTION OF HEAT AND SOUND

Rayleigh (1896) gave a clear physically based description of the interaction between sound waves and unsteady heat input. In essence, just as in any thermodynamic cycle, the addition of heat at high pressure leads to a net energy input, which is available to do work. If the energy gained from the unsteady combustion exceeds the rate of work done, the acoustic waves grow in amplitude.

Mathematically, for small amplitude disturbances in a simple combustor with a fluctuating rate of heat addition q within a volume V with bounding surface S, acoustic waves grow in amplitude if

$$\frac{(\gamma-1)}{\overline{\rho}\,\overline{c}^{\,2}}\int_{V}\overline{p'q}\,dV > \int_{S}\overline{p'\mathbf{u}}\cdot d\mathbf{S}$$

(1)

where p denotes the pressure. **u** is the flow velocity and we have assumed that the mean flow is negligible. ρ is the density, c the speed of sound and γ is the ratio of specific

heats. The overbar denotes a mean value and the prime a perturbation.

When the inequality in (1) is satisfied, the energy gained from the unsteady combustion exceeds the rate at which work is done on the surroundings, and the acoustic energy within the combustor gradually increases. As the amplitude increases the fluctuations in the heat input usually saturate, increasing less rapidly with amplitude than the energy loss term. A constant amplitude limit cycle is obtained when the left and right-hand sides of Eq. (1) balance.

The interaction between combustion and acoustic waves can therefore lead to instability in which linear disturbances grow into a nonlinear limit cycle. It also affects the frequency of oscillation. Again assuming that the mean flow is negligible, with uniform ρ and \overline{c} , a linear fluctuating rate of heat input/unit volume q leads small amplitude pressure disturbances p, which satisfy the inhomogeneous wave equation

$$\frac{1}{\overline{c}^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{(\gamma - 1)}{\overline{c}^2} \frac{\partial q}{\partial t}.$$
(2)

The term on the right hand side of Eq. (2) describes how the unsteady addition of heat generates a pressure perturbation.

If the unsteady heat addition were prescribed, then Eq. (2) could be solved to determine the resulting pressure perturbation. There would be an acoustic response to oscillations in q of any frequency, with a large amplitude response near the acoustic resonant frequencies of the combustor of volume V. However, self-excited oscillations occur because the rate of combustion is itself influenced by the unsteady flow. This feedback affects both the resonant frequencies of the system and whether they decay (stable) or grow (unstable) in time.

In many systems of interest, q is particularly influenced by changes in flow velocity or fuel-air ratio. We can illustrate the influence of this by considering the particular case of one-dimensional waves in a duct with one open and one closed end as illustrated in Fig. 1. Then the appropriate boundary conditions are



Fig. 1 Unsteady heat addition in a duct with an open and a closed end

As an illustrative example we consider the unsteady rate of heat input to be concentrated at the plane x = b. In lean combustion, the rate of heat addition increases as the fuel-air ratio is increased. In many LP systems, the fuel supply rate remains constant while the inlet air is modulated. Fuel-air ratio is then inversely proportional to inlet air velocity. We will investigate the effects of this through a model problem in which fluctuations in the instantaneous rate of heat release are equal to $-k u_1(t)$, where k is a positive constant.

Dowling (1995) and Dowling & Stow (2003) describe a way of solving the wave Eq. (2) with this form of unsteady heat release. The solution leads to the mode shapes and the resonant frequencies of the coupled duct/flame system. The lowest resonant frequency, for various values of k, is shown in Fig 2. For k = 0, the resonant frequency is of course the quarter wave resonance of the duct, $f_1 = c / 4\ell$. For larger values of k, the frequency is changed significantly. This effect was well understood by Rayleigh, who noted that a rate of heat input in quadrature ($\pm 90^\circ$) with the pressure fluctuations leads to frequency shifts.



Fig. 2 Variation of lowest resonant frequency with β , where β describes the sensitivity of the combustion to flow perturbation. The normalised frequency is f / f_1 and the rate of heat release/unit area is equal to $-\beta u_1(t)\overline{\rho}\overline{c}^2/(\gamma-1)$; $b = \ell/10$ (from Dowling & Stow 2003)

Combustion instability is therefore a genuinely coupled problem. Unsteady combustion generates acoustic waves, and within an acoustic resonator these linear perturbations can be sufficiently intense that they alter the rate of combustion. If the phase relationship is suitable, oscillations can grow in amplitude until limited by nonlinear effects. Both the acoustics and the unsteady combustion must be considered to understand and model this phenomenon. The coupling between them affects both the frequency of oscillation and the susceptibility to selfexcited oscillations.

MODELLING AND PREDICTION

Although the basic mechanism can be described simply, modelling the frequency and susceptibility to instability can be complex. The acoustics of the whole of the combustor from compressor exit to turbine entry have important influences on the frequency and mode shape of the self-excited oscillation. As we have seen the coupling between the unsteady combustion and the acoustic waves is also important. A range of approaches is needed to capture this phenomenon. Computational Fluid Dynamics (CFD), acoustic network models and experiment play complementary roles.

Of course, CFD can be used to model unsteady combustion, but there are difficulties associated with simulating combustion instabilities in this way. Firstly, the relevant geometry is extensive, since the computational domain needs to extend from compressor exit to turbine entry if the appropriate acoustics are to be included. High resolution and good sub-grid models are needed to capture the turbulence. Combustion occurs at sub-grid scales and, while there are models for diffusion flames and for some totally premixed flames, models for the partially premixed flames that occur in LP combustion are still at the development stage. The occurrence of a self-excited oscillation depends on the balance between the energy gained from unsteady combustion and that dissipated at the boundaries. We will discuss dissipative mechanisms in greater detail later, but here we note that they are often connected with the interaction between acoustic waves and the flow through cooling holes and other small gaps - just the features that are often poorly resolved by CFD. Only a limited frequency range of oscillation can be resolved by an uRANS (unsteady Reynolds-Averaged Navier Stokes) calculation, in which the turbulence is considered in a statistical way while the low-frequency coherent oscillations are resolved unsteadily. Such an approach can only be formally justified when the frequency of the turbulent oscillations is much higher than that of the selfexcited combustion oscillation. When these frequencies are more closely matched, it is necessary to use Large Eddy Simulation. Then both the turbulence and the combustion are treated as fully unsteady. Just as in an experiment, the turbulence leads to cycle-to-cycle variations and many cycles must be processed to determine the coherent combustion oscillation in the presence of the random turbulent fluctuations. Finally, we note that when CFD calculations are performed they are inevitably time consuming and that many periods of oscillation have to be calculated before the limit cycles to become established. In particular, continuing a CFD calculation until all periodic flow perturbations have decayed is not an efficient way of demonstrating that a particular operating condition is stable.

An alternative approach that we have found to be particularly advantageous way is to use detailed CFD in a section of the combustor just downstream of a premix duct to investigate the combustion response to unsteady inlet flows. This is commonly known as the 'Flame Transfer Function'. This can then be used in a network, or other acoustic analysis, to determine the overall system behaviour.

The Flame Transfer Function is generally a function of frequency, and needs to be investigated across a broad The amplitude and phase of this frequency range. combustion response is important in determining whether or not a self-excited oscillation occurs, and if it does, at what frequency. The combustion response at a single frequency can be investigated through harmonic excitation. However, this has the disadvantage that a long calculation time is required for starting transients to decay to give the periodic response to such an input. Short-duration pulses have the advantage that they contain a wide bandwidth of frequencies and their response can be quickly calculated. However, the combustion response is then contaminated by noise. Zhu et al (2001) have successfully developed an alternative approach in which pseudo-random noise is used as the input signal and System Identification techniques are used to determine the flame transfer function.

Experimental investigations of LP combustion instabilities are also challenging. If a rig is to have selfexcited oscillations representative of full-scale, it is important that the inlet and outlet planes should have the correct acoustic boundary conditions. In an annular combustor, the longest length is usually that around the circumference and so the lowest resonant frequencies are associated with azimuthal modes. This means that the instabilities in sector and full annular rigs will be quite different. Investigation of the flame transfer function is a way of getting useful information from sector rigs involving a single burner. The flame is then forced at a range of frequencies by acoustic waves generated upstream of the premixers. The excitation aims to modulate the airflow rate through the premixing ducts and could be achieved by using loudspeakers or, more practically, by a siren. Figure 3 shows results from such a forcing experiment. It is interesting to note that the form of the flame transfer function depends only weakly on operating pressure. A similar combustion response has been measured in an atmospheric rig (Riley 2003).



Simple models of the flame response in LP combustion systems are emerging from such experiments and from forced CFD calculations. When burning at lean conditions, the rate of combustion is very sensitive to fuel-air ratio. Indeed, since the flame is operating near its weak extinction limit any decrease in the air mass flow rate, making the flame richer significantly, enhances the rate of combustion. A simple flame transfer function for this system is therefore

$$\frac{\underline{Q}'(t)}{\overline{Q}} = k \, \frac{\phi'(t)}{\overline{\phi}} \,, \tag{4}$$

where Q(t) is the instantaneous rate of heat release and ϕ is the equivalence ratio (fuel-air ratio divided by the value for stoichiometric burning) of the mixture entering the combustor and k is a nondimensional number. If there were a uniform convection time τ from fuel injection to combustion, then

$$\frac{\phi'(t)}{\overline{\phi}} = -\frac{u_1'(t-\tau)}{\overline{u}_1} \,. \tag{5}$$

where $u_1(t)$ is the air velocity past the fuel bars. However, substitution of such an equivalence ratio into Eq. (4) would lead to a Flame Transfer Function with amplitude independent of frequency. That is clearly not the case in Fig. 3. Experimental data (Sattelmeyer 2000, Cheung *et al* 2003) and CFD (Armitage 2003) suggest that the response is better fitted by a model in which the time delay is taken to vary uniformly from $\tau - \Delta \tau$ to $\tau + \Delta \tau$. This leads to



where the circumflex denotes the complex amplitude at frequency ω . When the form for the unsteady equivalence ratio in (6) is used in Eq. (4), it gives a satisfactory fit to a range of experimental data. This variation in the time delay might be due to differences in the convective speeds across the premixing passages and between the convection time to the upstream and downstream edges of the combustion zone. Either way, we have a very simple description of the flame dynamics, which appears to be virtually independent of facility or ambient pressure provided appropriate time delays are used.

The geometries of typical industrial and aeroengine LP combustors are complex. The combustor may be cylindrical or annular and is connected to the compressor exit by a diffuser and premixers, which form a complicated geometry of interconnecting ducts. To predict self-excited oscillations it is necessary to be able to describe linear waves in such configurations. Calculation procedures with different levels of complexity are appropriate according to the mode of interest.



Fig. 4 The geometry of a particular industrial gas turbine combustor

Often it is the lowest few resonant frequencies that are most relevant. In cylindrical geometries these are associated with modes involving plane waves: all higher order modes are 'cut-off' at these frequencies and do not transport acoustic energy. In annular combustors, the lowest frequency modes are usually associated with circumferentially propagating waves, which vary axially but have little radial variation.

The propagation of these plane or circumferential waves in a straight section of duct can be described in a straightforward way. Application of appropriate boundary conditions then leads to the resonant frequencies and their mode shapes and their growth or decay rates. Comparison between theory and experiment is satisfactory, if the unsteady combustion is modelled appropriately, as shown for a simple test combustor shown in Figs 5 and 6.



Fig. 5 Variation of frequency with flow conditions, comparison of theory and experiment (from Kato et al 2002)



Fig 6 Variation of stability with flow conditions, comparison of theory and experiment (from Kato et al 2002)

For more complex systems of interconnected ducts, we have developed a network approach (Dowling 1999 and Stow & Dowling 2001). This approach is similar to an acoustic network analysis (Noreen 1997), but has been extended to include the other linear waves that occur in combustion systems, namely convected vortical and entropic disturbances. Consideration of the waves enables the development of the flow in straight sections of duct to be determined: if the form of plane or circumferential waves of frequency is known at one axial location, the

waves throughout the duct can be written down. The unsteady flow can be determined by adding up the contributions from all the linear waves. At the junctions of ducts of different cross-sectional areas or across the combustion zone, we convert the waves into flow perturbations and apply the appropriate continuity conditions to the fluxes of mass, momentum and energy, *e.g.* mass flow rate conserved, momentum flux increased by the axial force acting on the fluid and energy flow rate increased by the rate of combustion. The details are described in Dowling (1999) and Stow & Dowling (2001 and 2003). The network model is easy to use, can deal with real geometries like that in Fig 7, which including multiple flow paths and has been extended to incorporate acoustic dampers like Helmholtz resonators and cooling flows. However, it needs a Flame Transfer Function as an input.



Fig. 7 Typical aeroengine geometry

For higher frequency disturbances, the flow field becomes fully three dimensional. The linear perturbations can be resolved by a modal analysis. However, higher-order combustor modes in which the acoustic pressure varies with radius become coupled at the premixers (Akamatsu & Dowling 2001). It is then easier to use a linearised Euler calculation in the complex frequency domain (Armitage 2003).

CONTROL

For LPP systems the flame responds across a broad frequency range (see Fig. 3) and the plenum/combustor geometry has many resonances. It is therefore difficult to develop designs that are naturally stable at all these resonances across a broad range of operating conditions. Moreover, LP systems need to use most of the airflow to dilute the fuel in the premixing ducts if they are to achieve the low fuel-air ratios necessary for low emissions. This means that LP designs have less cooling airflows than conventional combustors, and so lose a valuable source of acoustic damping. Hence, it is usually necessary, and always advisable, to include either passive or active control in the combustors.

We can interpret the requirements for control through the generalised form of Rayleigh's criterion in Eq. (1). This showed that disturbances grow in amplitude if their net energy gain from the combustion is greater than the sum of their energy losses across the boundary:

$$\frac{(\gamma-1)}{\overline{\rho}\,\overline{c}^2}\int_V \overline{p'q}dV > \int_S \overline{p'\mathbf{u}} \cdot d\mathbf{S} \,. \tag{1}$$

To be effective, control needs to reverse the equality in Eq. (1) for small amplitude disturbances. This can be done either by increasing the energy loss at the boundaries or by decreasing the energy source term and can be implemented by either passive or active means.

One means of passive control is to modify the fuel input or the flame shape to reduce the energy source term on the right-hand side of Eq. (1). We have already seen that convection time delays from fuel input to combustion are important in determining the Flame Transfer Function. Times delays differing by half a period lead to cancellation of the combustion response. Significant reduction in the Rayleigh source term $\int_{V} \overline{p'q} dV$ can be achieved by designing the fuel system and combustion zone lead to

designing the rule system and combustion zone lead to elements of q' with different phases.

Passive acoustic dampers aim to increase the energy loss on reflection at the combustor boundaries, typically by converting energy in the acoustic wave into vorticity. This is usually done by attaching a resonator, for example a Helmholtz resonator or quarter-wave tube, to the walls of the combustor or by inserting perforated plates. When used in combustors, the neck of a Helmholtz resonator or the holes of a perforate need to be cooled by blowing cooler air through them. The interaction between acoustic waves and this bias flow leads to a very effective way of absorbing acoustic energy (Howe 1979). We have been developing models and design criteria to optimise the absorption. When the flow parameters are chosen appropriately over 80% of the incoming sound energy can be absorbed (Dupere & Dowling 2002, 2003), Eldredge & Dowling, 2003).

These passive techniques usually work well at modest and high frequencies but are less effective at low frequencies (< 200Hz, say) when tuned resonators become prohibitively large and the difference in time delays needed to reduce the Rayleigh source term may exceed the autoignition time delay. For that reason, interest has turned to active ways of reversing the inequality in Eq. (1).



Fig. 8 Schematic diagram of active control of combustion oscillations

A typical active control system is illustrated in Fig. 8. It consists of a sensor to monitor the oscillations in the combustor, a feedback system that determines the appropriate driving signal V(t) in response to the sensor signal S(t), and an actuator. The first demonstration of the effectiveness of active control was reported 20 years ago. Dines (1983) used a loudspeaker, driven in response to a measurement of unsteady heat release, to stabilise a laminar flame burning in a duct. However, it is not easy to generate effective acoustic sources at larger scale and so it is far more practical to seek to cancel the heat release fluctuations directly. Langhorne *et al* (1990) showed that modulations of the fuel supplied to a premixed flame could be used to stabilise a turbulent ducted flame. Such a procedure is remarkably easy to implement. Langhorne *et al* used a pressure signal to detect the perturbations, this was

amplified and time delayed and used to drive a fuel injection system. It was found that only small modulations in the fuel flow rate were required. The unsteady addition of 3% more fuel reduced the pressure band level within 3dB of the peak by 12dB, while the pressure band level below 400Hz was reduced by >5dB.

The work of Langhorne et al (1990) was supported by Rolls-Royce and publication was delayed to protect their commercial interests. Even before the results on the laboratory rig were published, Rolls-Royce and the Ministry of Defence had demonstrated the feasibility of combustion control on the afterburner of a full-scale RB199, although that work has only recently been released for publication (Moran et al 2000). The procedure was identical to that in the rig tests. A pressure signal was used to detect the phase of the oscillation, an appropriate time delay was applied and the shifted signal used to modulate the fuel flow rate. Once again, small levels of fuel modulation were found to be sufficient. The afterburner was able to achieve operating conditions impossible without control. It is therefore over ten years since the first full-scale demonstration of active control of combustion oscillations.

Of course, application to aeroengines places strong demands of safety-critical software and the emergence of combustion oscillations in land-based gas turbines has given new impetus to active control. Successful control has also been demonstrated on a full-scale industrial gas turbine (Seume *at al* 1998). But the current challenge is to give guarantees that the controller will not go unstable and cause harm and that it will be effective across all regimes of plant operation. That needs the integration of ideas from fluid mechanics, acoustics, combustion systems and control.

There are several approaches to the adaptive control of combustion oscillations. Billoud et al (1996) use the xfiltered LMS controller familiar from anti-sound applications. There is a conceptual problem it applying this method to instability control: the theoretical background assumes that the 'source' is independent of the control signal. That is usually true in a standard anti-sound application, but not for combustion instability control, where successful implementation will prevent the combustion oscillations from occurring and hence suppress the source. There have been successful demonstrations of LMS controllers applied to combustion instability (Billoud et al 1992, Kemal & Bowman 1996, Evesque & Dowling 2001), but no formal guarantees can be given of its convergence; the search for feedback filter coefficients that minimise the oscillations may lead to a more unstable condition or introduce new frequencies of oscillation. An alternative approach developed by Neumeier & Zinn (1996) involved a quick way of estimating the frequencies and amplitudes in the combustion oscillation, and then using a control strategy based entirely on Rayleigh's criterion. Fuel is added at these frequencies so as to lead to heat release 180° with the pressure. This requires known information on the time delays in the actuation system. Annaswamy et al (1998) developed low-order self-tuning controllers for simple combustion systems, There the controller structure and adaptive laws were based on the known open-loop behaviour of a particular combustor. The approach developed by Evesque et al (2000, 2003) build on this and seems promising. They investigated some general features of combustion within an acoustic resonator and showed that under some fairly general conditions, namely that

i) the amplitude of an acoustic wave reflected from the combustor walls less than that of the incident wave,

ii) the flame transfer function of limited band-width, *i.e.* the response tends to zero at high enough frequencies,

iii) the combustion itself stable, the instability arising due to between the combustion and the acoustics,

a particular form of low-order Self-Tuning Regulator could stabilise the system. They then developed an iterative scheme to find the controller coefficients adaptively. The update formula is Lyapunov-based, that is it constantly reduces a positive definite cost function, and so converges towards a stabilising set of parameters. The approach can accommodate time delays in the actuation system. The STR has been demonstrated on simulations (Evesque *et al* 2000), on small-scale burners (Evesque *et al* 2003) and on a laboratory-scale LPP burner (Riley *el al* 2003), and the results are very encouraging (see Fig. 8).



Fig 8 The effects of switching the STR on and off, showing the sensor signal, the actuation, and κ_1 a filter coefficient determined adaptively (from Riley et al 2003)

CONCLUSION

Environmental concerns are causing gas turbine combustors to operate close to instability where self-excited oscillations occur and present a significant challenge to gas turbine manufacturers. The instability is due to coupling between the acoustic waves and the combustion – the unsteady combustion generates sound and the resulting velocity and fuel-air ratio fluctuation perturb the flame still further. Acoustic absorbers should be incorporated in LP combustors, and for low frequency oscillations active control has enormous potential.

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