Proceedings of the International Gas Turbine Congress 2003 Tokyo
November 2-7, 2003

IGTC2003Tokyo TS-016

Dynamics of Asymmetric Rotors using Solid Models

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ABSTRACT

Asymmetric rotors or shafts with dissimilar moments of area form a special category of rotor dynamics problems. Of specific interest for these rotors is the instability in the region of half critical speed, also called gravity critical speed.

General analysis of such rotors is limited to Jeffcott type beam models. This involves in finding the asymmetry parameter of the shaft DK/K, and use modal properties or simple physical model of a disk on mass-less shaft with asymmetry for a Jeffcott type of analysis.

Recent trends in rotor dynamics analysis indicate a shift from beam to solid models, which enable the classical finite element approach to be used in the design process. The rotor geometry is maintained without any simplification to obtain beam models; first CAD model is made, meshed using appropriate elements and perform the rotor dynamics analysis. In this paper, we demonstrate the procedure for rotor dynamics of a solid asymmetric rotor without beam model simplification.

INTRODUCTION

Almost all the rotors are circular in cross-section, i.e., they are symmetric shafts and have same lateral stiffness in any plane of bending. Until recently, the most common practice in rotor dynamic analysis is to make a one dimensional model, beam type for bending or lateral motion and twisting rod type for angular or torsional vibrations. Depending on the advances in the computational front, tabular methods, transfer matrix methods and finite element methods are adopted, see Rao (1996).

Asymmetric Shafts

For generator rotors on horizontal shafts, besides the main critical speed, disturbing whirl amplitudes have been observed at half critical speed. On vertical shafts, such is not the case indicating that gravity is one of the causes for this. The effect is observed more clearly with shafts of rectangular cross-section, which is not equally stiff in all directions as a circular shaft. In practice rotors of turbogenerators, because of their construction, exhibit a marked difference between the stiffnesses in two principal directions. The disturbance that occurs at speeds equal to half the critical speed of the rotor, can take place for even a perfectly balanced disk and therefore this disturbance is due to the instability condition, rather than due to unbalance.

For Jeffcott type rotors with cross-section having two principal axes about which the second moments of area are maximum and minimum with spring constants, K + DK and K – DK, with an average value K, the stiffness variation can be taken K + DK sin 2wt.

Manuscript Received on July 7, 2003

The equation of motion then turns out to be Mathieu type, see Hayashi (1965) and in the absence of gravity, its stability characteristics can be obtained using Floquet's theory.

Timoshenko (1955) and Den Hartog (1956) assumed the stiffness to be constant, minimum or maximum for a period p/2w and connected the linear solutions using appropriate boundary conditions to study the stability characteristics in the presence of gravity. Tondl (1965) used a rotating coordinate system and studied the stability boundaries of shafts with dissymmetry.

The bearing stiffness plays an important role in the dynamic behavior of heavy rotors. Typically, hydrodynamic bearings are asymmetric in nature and cross-coupling between two orthogonal axes for stiffness; these stiffness coefficients affect the stability due to oil whirl. In practice it is important to consider the influence of hydrodynamic bearings on the asymmetric rotating shafts, as in the case of alternator rotors in turbogenerator sets. When the asymmetric shaft on rigid bearings was considered it is easy to write the equations in rotating coordinates. However, when the bearing forces are to be combined to the asymmetric shaft, it becomes useful to write the equations in fixed coordinate system. For Jeffcott models, Newmark average acceleration time marching technique was adopted by Rao and Bhaskara Sarma (1986) to determine the transient whirl response from an initially displaced rotor at a constant speed and thus assess the stability of asymmetric shaft rotors mounted on fluid film bearings.

A few more relevant papers for asymmetric shafts can be cited to be: Taylor (1945), Crandall and Brosens (1961), Yamamoto and Kono (1971), Iwatsubo (1971), Forrai (1976), Inagaki, Kanki and Shiraki (1979).

By and large, asymmetric rotor modeling and analysis is limited to one dimensional beam models of Jeffcott type and this paper attempts to outline a procedure whereby CAD models can be directly used for rotor dynamic analysis using solid elements.

Solid Model Analysis

One-dimensional models discussed above in rotor dynamic analysis suffer from many disadvantages, viz.,

1. Real life rotors are not one-dimensional.
2. Considerable time and effort are involved in deriving good 1-D beam model from actual drawings of rotors.
3. The influence of disks on shafts, vice versa is not accounted
4. Centrifugal effects of distributed shafts and mounted parts cannot be accounted.
5. Gyroscopic effects are calculated as separate elements of equivalent disks and given as inputs to the beam model; otherwise the spin speed does not enter into the determination of Campbell diagrams and critical speeds.

Though the need for using solid elements of the rotor was stated by Bellamy et al. (1985), an application of rotor dynamic analysis using solid models was not made till recently. Stephenson and Rouch (1993) modeled rotating shafts using cubic axi-symmetric finite elements by including gyroscopic skew-symmetric matrix. Yu et al (1999) used 3-D solid finite elements for an orbiting shaft. The shafts are defined by super-elements consisting of four basic continuum elements, whose geometry is a quarter-annulus. Tapered hollow-shaft results of Stephenson and Rouch were considered. The model developed included torsional as well as axial modes. Rao, Sreenivas and Veeresh (2002) presented an application for Jeffcott type solid rotors with three disks by developing special macros to adopt ANSYS structural analysis software for rotor dynamics analysis. The results obtained showed close agreement with the beam models for both forward and backward whirl modes considering the gyroscopic influence in the beam model. They developed further the rotor dynamics application to determine the natural frequencies and mode shapes of two spool rotor systems. Both the centrifugal stiffening and spin softening effects were included. It was shown that the backward whirl is predominantly affected due to spin softening effect which disappeared at a spin speed equal to the natural frequency of the stationary shaft. The forward whirl was influenced by centrifugal stiffening effects and the frequencies are slightly higher than the corresponding beam model results.

An accurate rotor dynamic analysis needs solid models for the rotors. In modern day designs, all components are CAD modeled and auto-meshing features in commercial codes makes meshing very accurate and accomplished in very little time compared to procedures in beam modeling. Rao (2002) showed that rotor dynamics can now be accomplished by solid element models and eliminate the beam model disadvantages mentioned above. However, the CPU time, RAM and hard disk requirements go up considerably. With recent advances in computers, these limitations are eliminated, thus making solid rotor dynamics more attractive. Recently, Rao and Sreenivas (2003) discussed the interaction of support structures on the dynamics of a dual rotor system. The system considered is a dual rotor, supported on flexible bearings, which are in turn mounted in a flexible casing. The rotors are modeled using solid elements. The bearings are simulated as springs, wherein the direct and cross coupled stiffness and damping coefficients are applied. The casing is also modeled and meshed using solid elements.

In this paper, we will present transient rotor dynamic analysis of asymmetric shafts modeled using solid elements that take into account stress stiffening and spin softening effects.

TRANSIENT ANALYSIS OF SYMMETRIC BEAM TYPE JEFFCOTT ROTORS

Solid rotor model analysis under steady conditions is established by Rao, Sreenivas and Veeresh (2002) for Jeffcott type rotors and the results are validated from established beam analyses. The analysis there is carried out using specially written macros in ANSYS (2001). Here we consider the transient rotor dynamics of beam models in a similar manner to study the orbital response. The model chosen is from a beam model given by Gunter, Choy and Allaire (1978) and a closed form analysis for steady state orbits is established, see Rao (1982, 1996).

The rotor having a mass 54.432 kg, with its rigid bearing critical speed equal to 4820 rpm, is mounted on two 4 axial groove bearings 2.54 cm dia and 1.27 cm long, with a radial clearance 0.00254 cm and viscosity at operating temperature, 0.0242 N sec/m^2. The shaft stiffness is therefore $K = Mp^2$ = 1.387 E07 N/m. The stiffness coefficients for the bearing at 4500 rpm, Sommerfeld Number S = 0.5488 are

K_{zz} = 4.16 E07 N/m; K_{yy} = 1.01 E07 N/m;
K_{zy} = 3.12 E07 N/m; K_{yz} = 4.16 E05 N/m;

This rotor has two natural frequencies, 3685 rpm and 4476 rpm. It is also shown that the steady state orbit is forward whirl for speeds below the first critical speed 3685 rpm and above the second critical speed 4476 rpm. In between the two critical speeds the whirl is backward in nature.

The rotor is modeled in ANSYS using Beam 44 elements and Mass 21 elements. The bearings are simulated by COMBIN-14 elements. The rotor is disturbed from the origin and the orbit obtained in transient condition is plotted until steady state conditions are reached.

The results for 3000 rpm are compared in Fig. 1 with steady state solution and it can be seen that the transient solution predicts the final orbit accurately. Here, the rotor is in forward whirl.

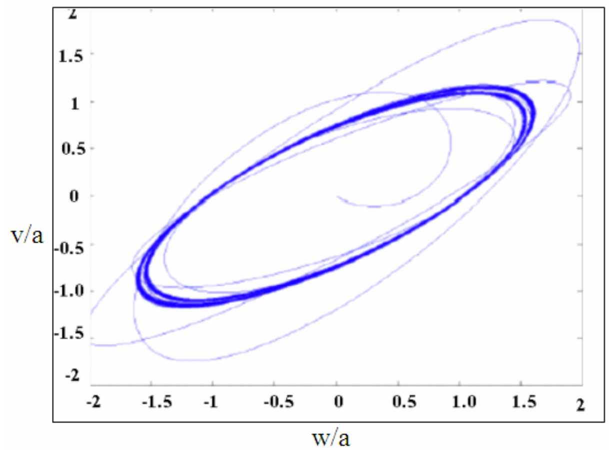

Fig. 1a Steady State Orbital Response from Transient Analysis

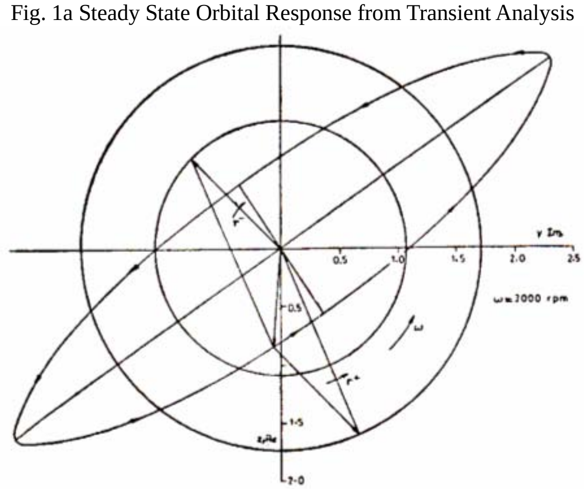

Fig. 1b Steady State Closed Form Orbital Solution at 3000 rpm

Between the two critical speeds, the rotor has a backward whirl and the results at 4000 rpm are given in Fig. 2. The final orbit obtained in transient analysis matches with the steady state solution. To ascertain the nature of the whirl, Fig. 3 is plotted showing the initial orbit and the final orbit stages – it is seen clearly that the orbit begins as a forward whirl, however, as steady conditions are approached, backward whirl sets in.

The transient solution at 6000 rpm, beyond the second critical speed is depicted in Fig. 4 and compared with the steady state solution. We notice that there is a good match between the results. This establishes the procedure developed on ANSYS for carrying out transient rotor dynamic analysis.

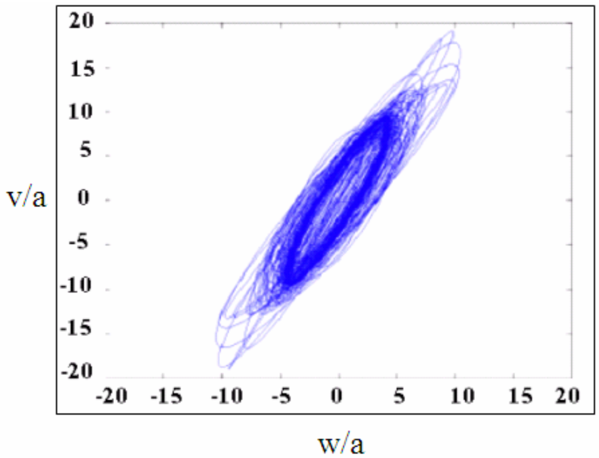

Fig. 2a Steady State Orbital Response from Transient Analysis

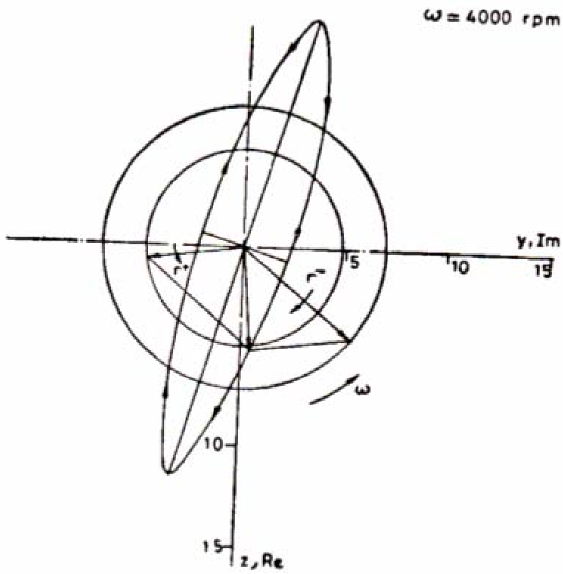

Fig. 2b Steady State Closed Form Orbital Solution at 4000 rpm

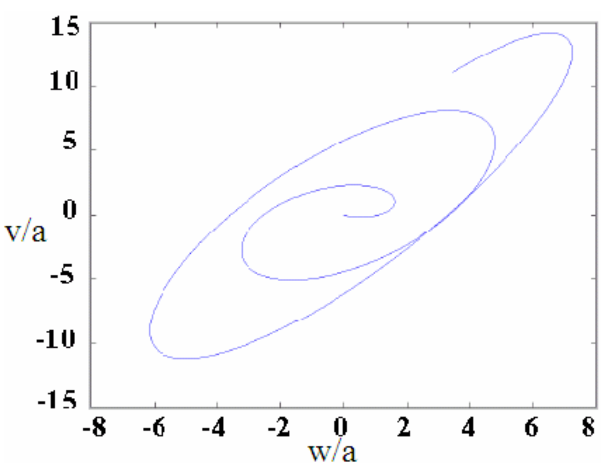

Fig. 3a Initially the Whirl begins as Forward at 4000 rpm

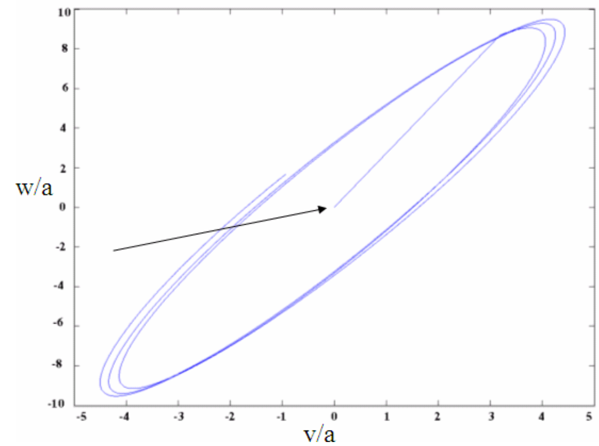

Fig. 3b Backward Whirl sets in during the Final Stage of Approaching Steady State Orbital Response at 4000 rpm

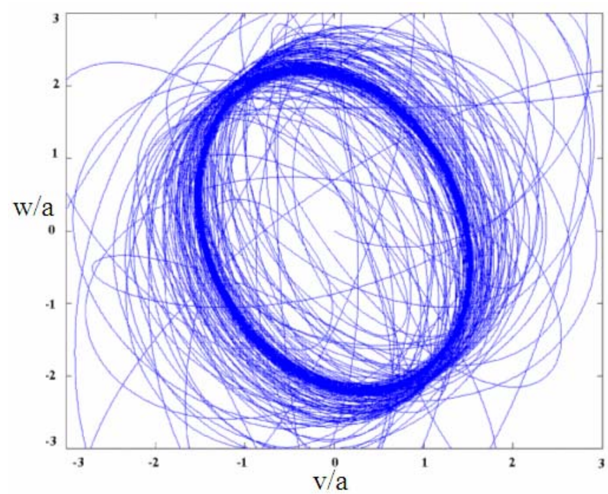

Fig. 4a Steady State Orbital Response from Transient Analysis

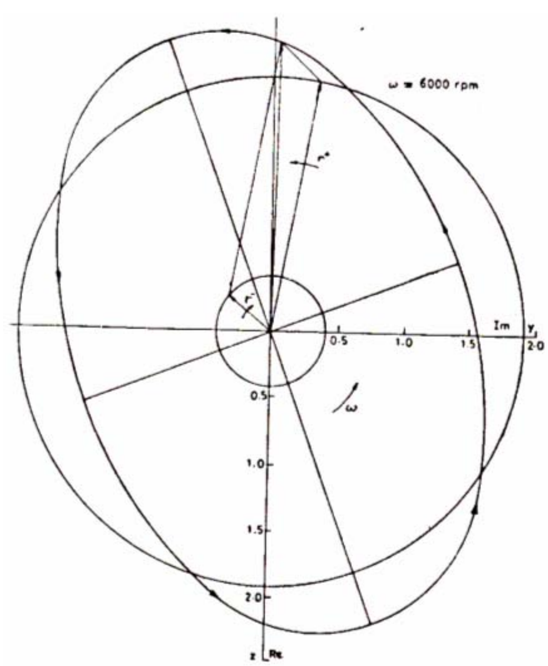

Fig. 4b Steady State Closed Form Orbital Solution at 6000 rpm

INFLUENCE OF STRESS STIFFENING AND SPIN SOFTENING

Conventional beam models do not account for stress stiffening and spin softening effects, as there is no cross-sectional dimension in the analysis. These effects are generally considered in turbomachine blade dynamics, the derivation of these terms is given by Carnegie (1967). In beam rotor dynamic models, the spinning effects cannot be included even when there are disks on the shafts. These effects can be easily incorporated in solid element; Rao (2002) demonstrated using a dual rotor model with Beam44 elements with capability of taking solid and hollow circular cross-sections. A dual rotor system is generally employed in aircraft engines to save space and keep the weight to a minimum by having a hollow outer spool which mounts the high pressure compressor and turbine running at relatively higher speed through which an inner spool rotor mounts the low pressure compressor and turbine rotors. Such an arrangement is used here where centrifugal effects can play significant role in stress stiffening and spin softening effects. The bearings are simulated by COMBIN-14 elements. Both the rotors are subjected to spin; the two different speeds are given by using ANSYS special feature to adopt subroutine “useracel”. The fast and slow components of elements as required by this subroutine were created. By default, this model takes spin-softening effect. Mass21 elements are adopted for simulating the disks. Total number of elements is 80 and the nodes are 76. The model is shown in Fig. 5. The Campbell diagram obtained for this model is given in Fig. 6.

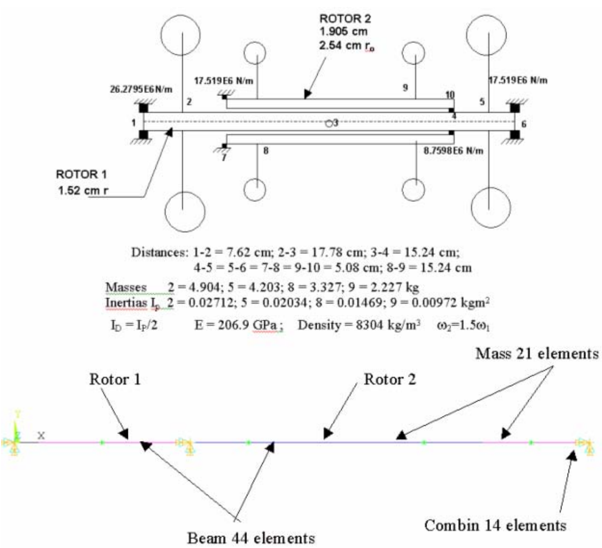

Fig. 5 Twin Spool Beam Model with Spin Softening Effects

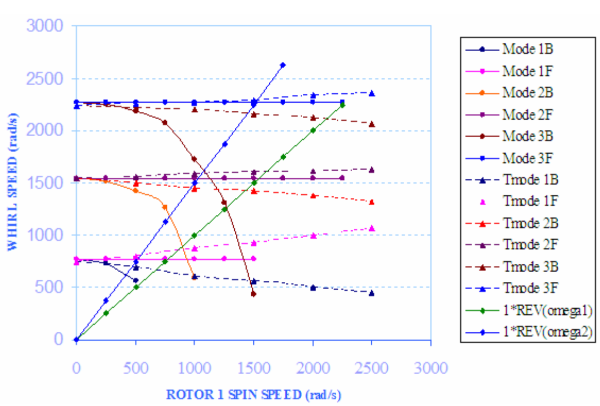

Fig. 6 Campbell Diagram including Spin Softening Effects

Since the model is one-dimensional, there is no stress-stiffening effect that raises the natural frequencies with speed. The eigen value problem with spin softening effect is defined by

$$\left|\left([K]-\omega_s^2[M]\right)-\omega^2[M]\right|=0 \qquad \left|[\bar{K}]-\omega^2[M]\right|=0$$

where $[K]$ is stiffness matrix, $[M]$ is the mass matrix, w is the natural frequency and w_s is the spin speed, and the effective stiffness matrix is denoted by a bar above K.

For backward whirl, the natural frequency decreases with spin speed and the effective stiffness becomes zero when the spin speed becomes the natural frequency of the stationary shaft. Therefore the backward whirl natural frequency drops at a faster rate when spin softening is accounted for and disappears at a spin speed equal to the natural frequency under stationary conditions. Conventional beam models do not predict this aspect of backward whirl frequency. The unbalance response of a system with spin softening effect is therefore considerably affected. This effect will be demonstrated in the following section.

TRANSIENT ANALYSIS OF SYMMETRIC SOLID JEFFCOTT ROTORS

Rao and Bhaskara Sarma (1987) developed a transfer matrix method with the state vector quantities as general functions of time, to determine transient orbital response of beam type rotors mounted on hydrodynamic bearings. The results of this time dependent analysis are used here to compare the transient analysis of symmetric solid rotor models.

In all solid model analyses, the rotor system was meshed using SOLID45 elements of ANSYS. The unbalance load was applied to the midspan of the rotor models. For these transient studies, the unbalance excitations were generated from MATLAB (with a time period of 0.0005 sec, which satisfies the Nyquist criterion) and these excitations, were read in from a text file and applied onto the ANSYS model. The rotor was supported on the bearings, which had the appropriate stiffness and damping coefficients in them. The rotor was spun at the appropriate speed and the unbalance excitations were applied onto them, from the text file.

The transient dynamic analysis was solved by Full method of ANSYS (Direct Integration method) and the response in time domain was obtained. The response in two directions of bending were read into an array and plotted against each other, to obtain the orbital response of the rotor system for the applied excitations.

Fig. 7 shows the rotor as a solid model with length 242 mm. 8 noded brick elements are used. The unbalance load is 0.544E-3 Kg-m giving an eccentricity of 0.01E-03 m. Stiffness values at 3000 rpm are:

K_{zz} = 0.175 E05 N/mm; K_{yy} = 0.175 E05 N/mm;
K_{zy} = 0.1 E05 N/mm; K_{yz} = 0.1 E05 N/mm.

A damping ratio 0.1 was used for quick convergence.

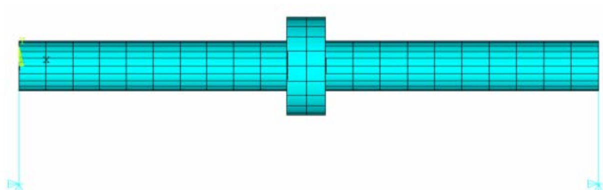

Fig. 7 Solid Rotor Model adopted for Transient Analysis

The results of the transient analysis of beam as well as solid models are given in Fig. 8. They are in good agreement.

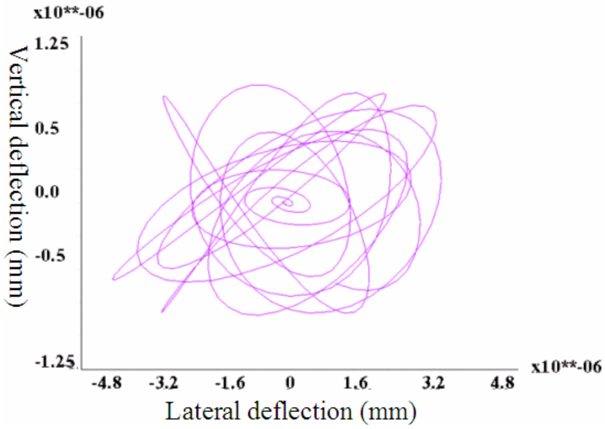

Fig. 8a Solid Rotor without Spin Softening at 3000 rpm

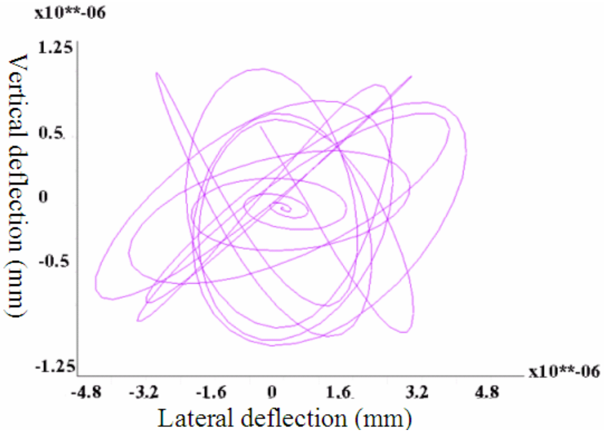

Fig. 8b Transient Analysis of Beam Model at 3000 rpm

When the spin softening effect is included, the unbalance response is significantly affected, see Fig. 9. Though the nature of the orbit remains similar, the spin softening enhances the amplitude considerably.

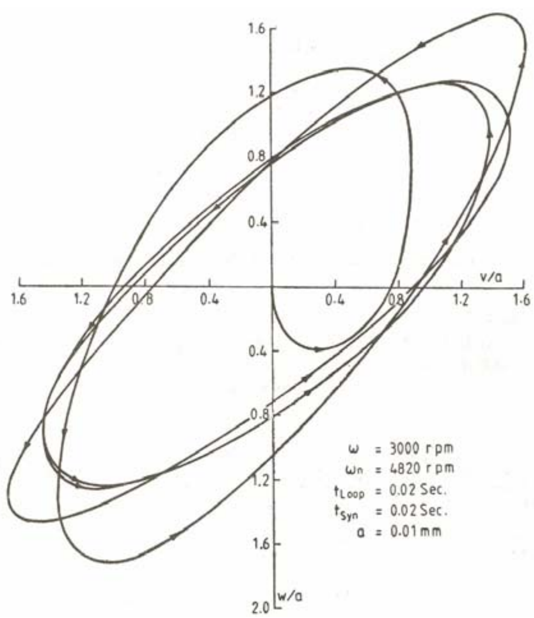

Fig. 9a Transient Analysis Results of Beam Model at 3000 rpm

TRANSIENT ANALYSIS OF ASYMMETRIC ROTORS

Rao and Bhaskara Sarma (1986) developed a transient response analysis of beam type Jeffcott rotors using Constant Average Acceleration Newmark method. They considered a rotor of mass 54.3 kg with a rigid bearing critical speed 4820 rpm. The asymmetry of the shaft considered was DK/K = 0.4. Both the support bearings were taken as rigid. The response due to gravity alone and unbalance alone was reported. The same rotor is considered here to determine the transient response.

The response peaks at 0.46 times the critical speed due to gravity and at 0.77 and 1.16 times the critical speed due to unbalance. Here, we report the results at the 0.46 and 0.77 times the critical speed to obtain the final steady state response and the results are compared in Figs. 10 and 11.

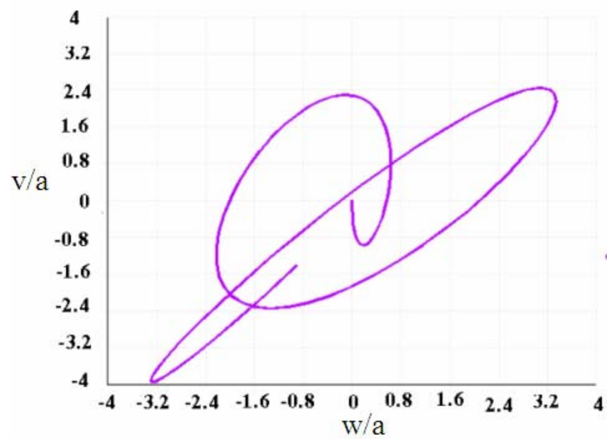

Fig. 9b Results of Solid Model with Spin Softening at 3000 rpm

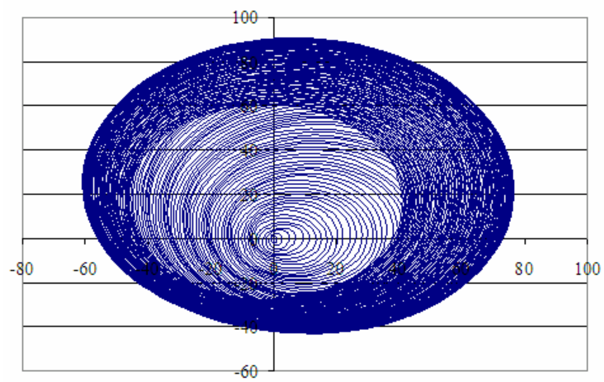

Fig. 10a Transient Analysis Results of an Asymmetric Solid Rotor at Gravity Critical Speed

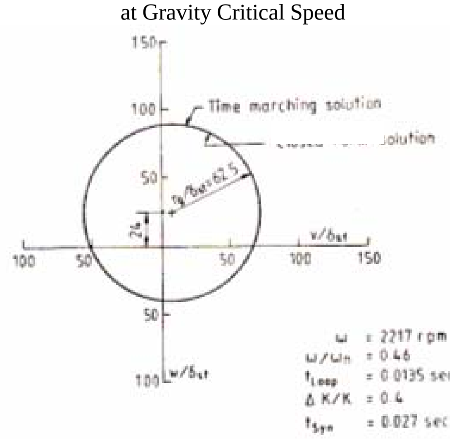

Fig. 10b Transient Analysis Results of an Asymmetric Beam Rotor at Gravity Critical Speed

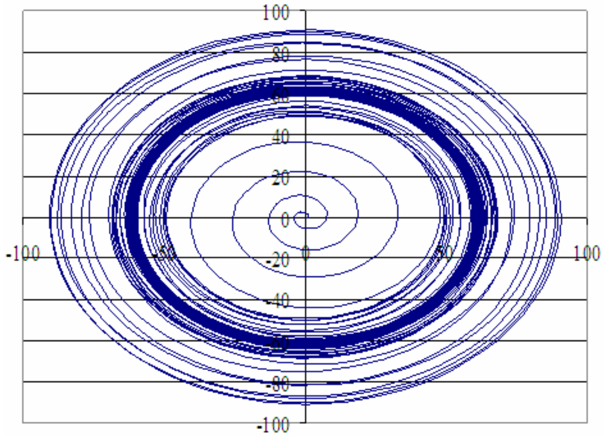

Fig. 11a Transient Analysis Results of an Asymmetric Solid Rotor at First Critical Speed due to Unbalance

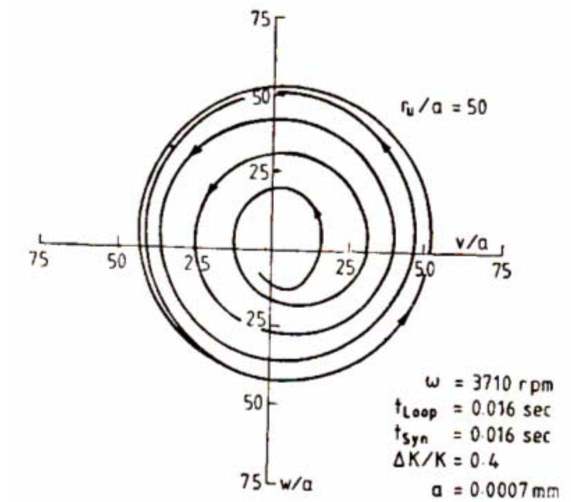

Fig. 11b Transient Analysis Results of an Asymmetric Beam Rotor at First Critical Speed due to Unbalance

CONCLUSIONS

A transient rotor dynamic analysis procedure is demonstrated that enables a solid rotor analysis of asymmetric rotors. The advantages in adopting this procedure are:

1. Real life rotors are not one-dimensional and CAD models of rotors can be directly used. This enables saving of considerable time and effort in deriving good 1-D beam models from actual drawings of rotors.
2. The influence of disks on shafts, vice versa can be accounted.
3. Centrifugal effects of distributed shafts and mounted parts are included. Stress stiffening as well as spin softening is included which have significant effect on the critical speeds and unbalance response.
4. Gyroscopic effects are calculated as separate elements of equivalent disks and given as inputs to the beam model; otherwise the spin speed does not enter into the determination of Campbell diagrams and critical speeds. These effects are automatically included in solid models.
5. The rotor dynamics analysis can now predict the whirl amplitudes more accurately, rather than just estimating the critical speeds and unstable regimes, thus improving the design capabilities of generator asymmetric rotors.

ACKNOWLEDGEMENTS

The authors are grateful to QuEST administration in carrying out this work and allow a publication of the results.

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